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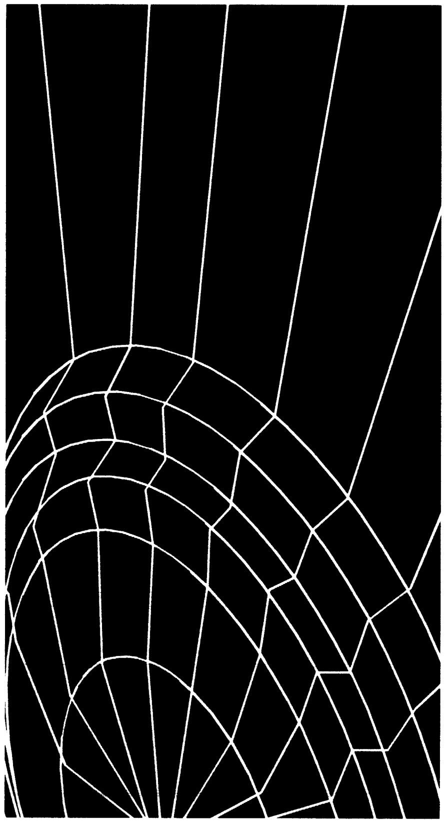

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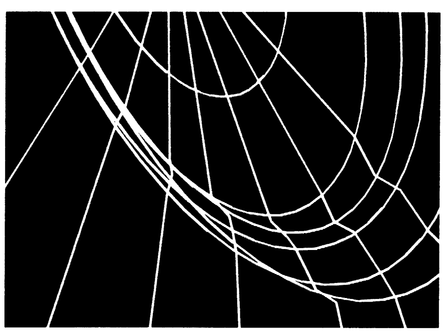

Volume E

Demonstration Problems

Version K7

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Document Title: **MARC Volume E: Demonstration Problems, Part V, Version K7**
Part Number: RF-3018-07.V
Revision Date: June, 1998

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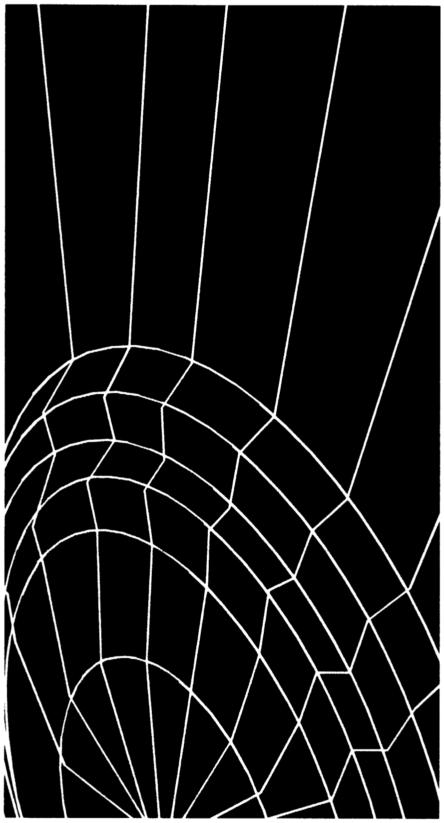

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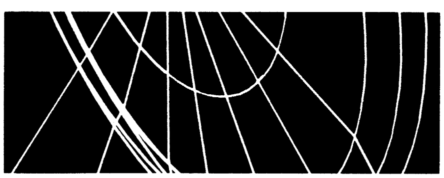


Volume E

Demonstration Problems

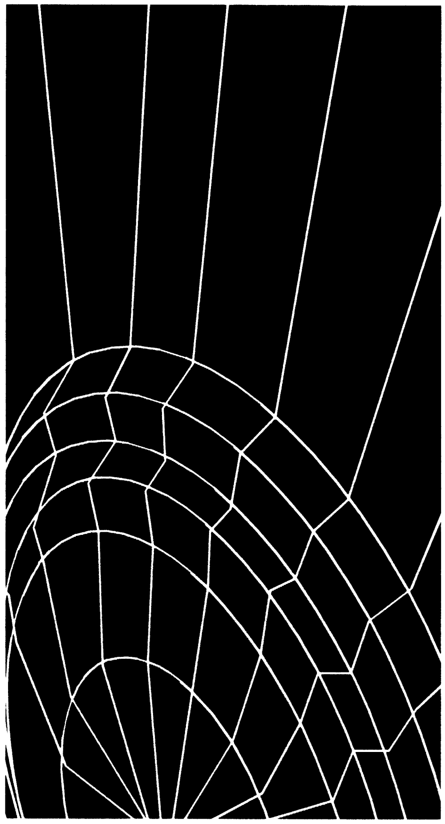
Version K7

Part V

- Fluids
- Design Sensitivity and Optimization

Part V

Volume E: Demonstration Problems



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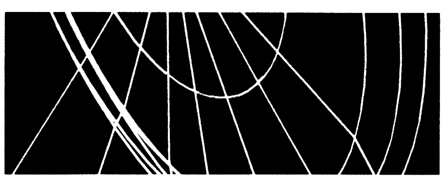

Volume E

Demonstration Problems

Version K7

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Chapter 9
Fluids

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9 Fluids

MARC provides a variety of analysis capabilities based on Navier-Stokes equations for viscous, incompressible fluid mechanic applications. A discussion on the use of fluid mechanics analysis can be found in *MARC Volume A: Theory and User Information*. The summary of the features is given below:

Selection of Element Topology:

- 2D triangular and quadrilateral
- Axisymmetric triangular and quadrilateral
- 3D tetrahedral and hexahedral

Choice of Element Formulation:

- Mixed method
- Penalty method

Selection of Material Behavior:

- Newtonian (linear) fluid material
- Non-Newtonian (shear-rate dependent viscosity) fluid material

Option for Multi-Physics Coupling:

- Fluid-Thermal
- Fluid-Solid
- Fluid-Solid-Thermal

Compiled in this chapter are a number of solved problems. Table 9-1 summarizes the element type and options used in these demonstration problems.

Table 9-1 Fluids Demonstration Problems

Problem Number	Element Type(s)	Parameters	Model Definition	History Definition	User Subroutines	Problem Description
9.1	11 27	DIST LOADS	DIST LOADS STEADY STATE	—	—	Planar Couette flow.
9.2	10 28	DIST LOADS	DIST LOADS STEADY STATE	–	—	Poiseuille flow.
9.3	11 6	—	STEADY STATE	—	—	Fluid squeezed between two plates.

Table 9-1 Fluids Demonstration Problems (Continued)

Problem Number	Element Type(s)	Parameters	Model Definition	History Definition	User Subroutines	Problem Description
9.4	11	—	STEADY STATE	—	—	Driven cavity flow.
9.5	11 27	—	INITIAL VEL	TRANSIENT NONAUTO	—	Flow past circular cylinder.
9.6	11	FLUID THERMAL	STEADY STATE	—	—	Flow over multiple steps in a channel.
9.7	11	DIST LOADS FLUID THERMAL	DIST LOADS STEADY STATE	—	—	Natural convection.
9.8	11	DIST LOADS	DIST LOADS STEADY STATE	—	—	Flow around tubes.

9.1 Planar Couette Flow

This is an example of simple shear flow of a viscous fluid between the parallel surfaces. A steady state analysis is performed. The results can be compared with the exact analytical solution.

This problem is modeled using three techniques summarized below:

Data Set	Element Type(s)	Number of Elements	Number of Nodes	Differentiating Features
e9x1a	11	24	35	Mixed method
e9x1b	27	6	29	Mixed method
e9x1c	27	6	29	Penalty method

Element

Element type 11 is a lower-order, 4-node, planar element using bilinear interpolation. Element 27 is a higher-order, 8-node, planar element using biquadratic interpolation functions.

When element types 11 and 27 are used in the mixed formulation, each node has two velocity degrees of freedom and one pressure degree of freedom.

When the penalty formulation is used, only the two velocity degrees of freedom are at each node. The penalty factor is entered via the PARAMETER model definition option.

Model

The two surfaces are 1.2 inches apart and the length is 2.0 inches. The meshes used are shown in Figures 9.1-1 through 9.1-3. Only the upper-half of the domain is discretized due to symmetry.

Boundary Conditions

It is assumed that parallel flow will develop; hence, along the inlet and outlet side the boundary conditions are $V_y = 0$. On the bottom surface, due to symmetry, $V_y = 0$. The top surface is considered to be moving with velocity $V_x = 1.0$. There is no relative velocity of the fluid and the surface. This is defined through the FIXED VELOCITY option.

Material

The material is a Newtonian fluid with a viscosity of 1.0 lbf. sec/square inch and a mass density of 1.0 lbs/cubic inch.

Results

The velocity profile is shown in Figure 9.1-1 through Figure 9.1-3 and is identical for all three element types used. Comparison of computation and analytical result is given in Table 9.1-1 and Figure 9.1-4.

Table 9.1-1 Comparison of Fluid Velocity Obtained from Finite Element Computation against Analytical Result

Vertical Distance	e9x1a	e9x1b	e9x1c	Analytical
0.000000e+00	1.000000e+01	1.000000e+01	1.000000e+01	1.000000e+01
1.000000e-01	9.750000e+00	9.750000e+00	9.749950e+00	9.750000e+00
2.000000e-01	9.000000e+00	9.000000e+00	9.000000e+00	9.000000e+00
3.000000e-01	7.750000e+00	7.750000e+00	7.749950e+00	7.750000e+00
4.000000e-01	6.000000e+00	6.000000e+00	6.000000e+00	6.000000e+00
5.000000e-01	3.750000e+00	3.750000e+00	3.749950e+00	3.750000e+00
6.000000e-01	1.000000e+00	1.000000e+00	1.000000e+00	1.000000e+00

Parameters, Options, and Subroutines Summary

Examples e9x1a.dat, e9x1b.dat, and e9x1c.dat:

Parameters	Model Definition Options
DIST LOADS	CONNECTIVITY
ELEMENTS	CONTINUE
END	COORDINATES
FLUID	CONTROL
SIZING	DIST LOADS
	END OPTION
	FIXED VELOCITY
	ISOTROPIC
	NO PRINT
	POST
	STEADY STATE

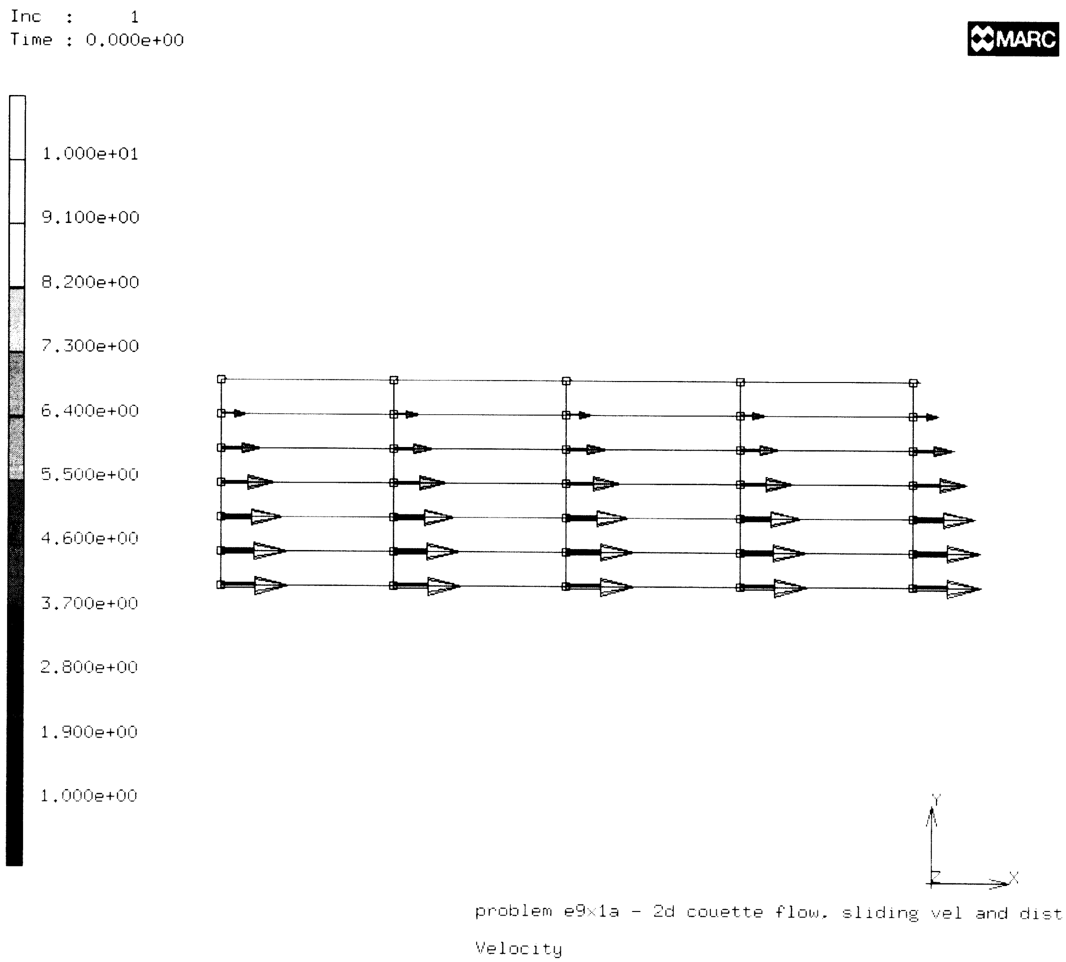


Figure 9.1-1 Vector Plot of the Couette Flow Velocity Field, Discretized using Element Type 11 and the Mixed Method

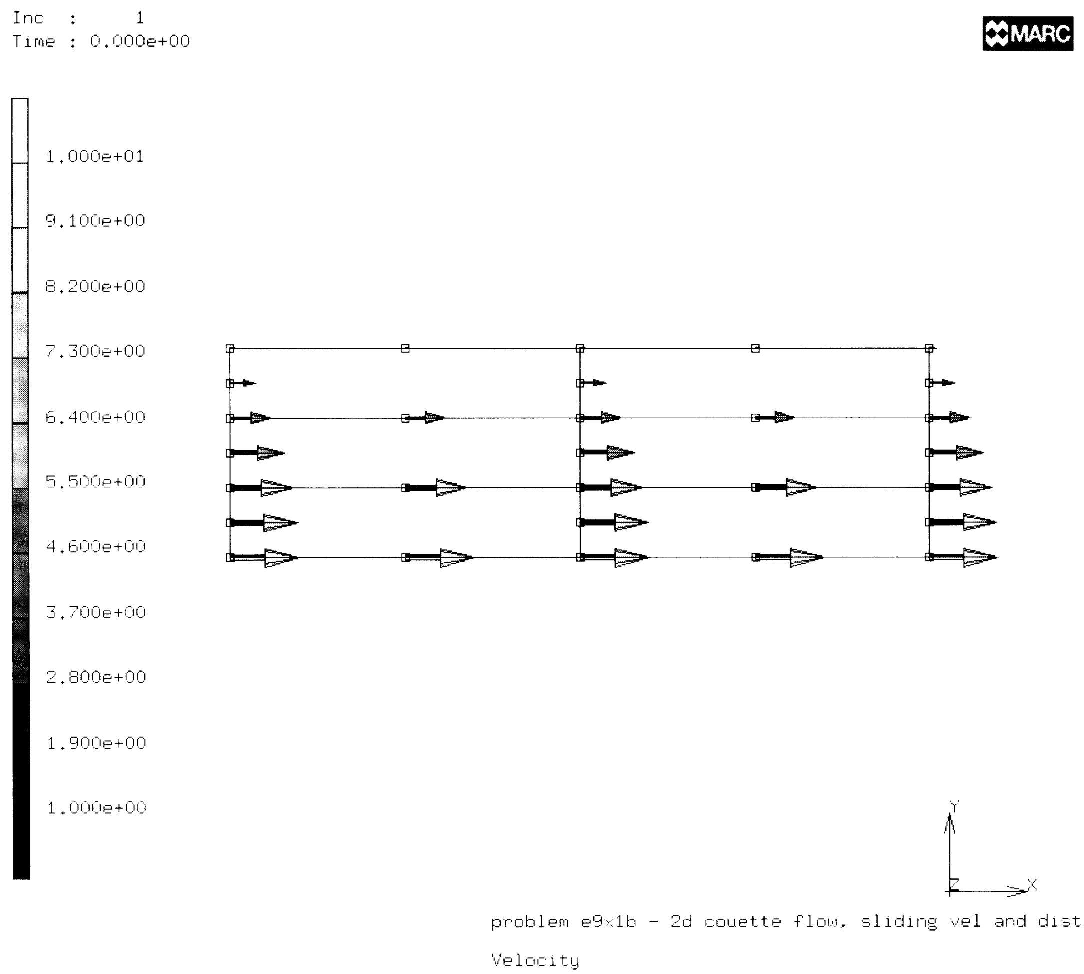


Figure 9.1-2 Vector Plot of the Couette Flow Velocity Field, Discretized using Element Type 27 and the Mixed Method

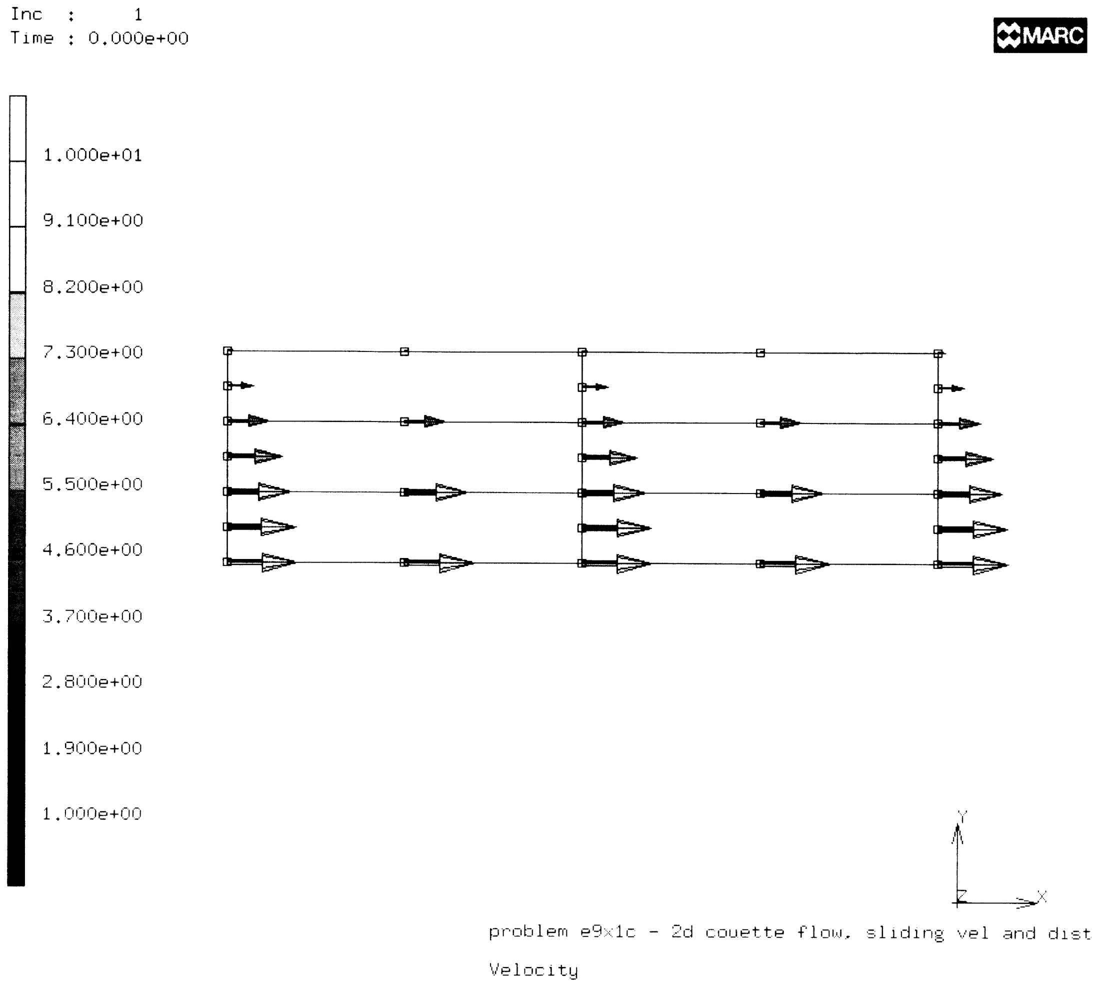


Figure 9.1-3 Vector Plot of the Couette Flow Velocity Field, Discretized using Element Type 27 and the Penalty Method

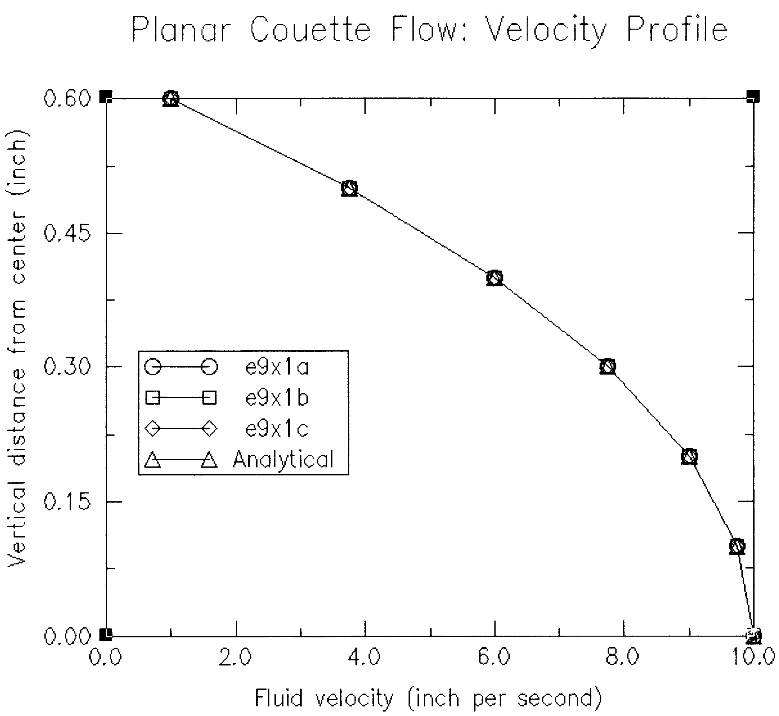


Figure 9.1-4 Comparison of Computation and Analytical Results

9.2 Poiseuille Flow

This problem demonstrates the steady state solution for a viscous fluid in a circular pipe. A pressure gradient is applied along the length of the pipe. The flow is assumed to be axisymmetric and, because the pipe is infinitely long, there will be no variation along the axis in steady state flow.

This problem is modeled using the three techniques summarized below:

Data Set	Element Type(s)	Number of Elements	Number of Nodes
e9x2a	10	24	35
e9x2b	10	48	63
e9x2c	28	24	93

Element

Element 10 is a 4-node, axisymmetric element using bilinear interpolation. Element 28 is an 8-node, axisymmetric element using biquadratic interpolation. The mixed formulation is used for all the above stated problems. Each node has two velocity degrees of freedom and a pressure degree of freedom.

Model

The radius of the pipe is 1.0 inch and the length is 3.0 inch. The finite element models are shown in Figures 9.2-1 through 9.2-3 for the different mesh density and/or element types.

Boundary Conditions

An axisymmetric analysis is performed; hence, along the line r = 0, the $V_r = 0$. At the outer radius is the rigid wall. No-slip boundary conditions require the fluid velocity on the wall to be equal to zero, so $V_r = V_z = 0$. The radial velocity is considered to be zero at the inlet (Z = 0) and outlet (Z = 3). A pressure gradient is applied by specifying a stress of 1 psi on the inlet surface.

Material

The fluid is Newtonian with a viscosity of 1.0 lbf/square inch and a mass density of 1.0 lb/cubic inch.

Results

The solution of this problem can be found in any text book on fluid mechanics. The steady state distribution of the axial velocity is:

$$V_z = -\frac{1}{4\mu}\frac{dp}{dz}(R^2 - r^2)$$

The MARC calculated solution for the different models is given in Figures 9.2-1 through 9.2-3. Comparison of computation and analytical result is given in Table 9.2-1 and Figure 9.2-4.

Table 9.2-1 Comparison of Fluid Velocity Obtained from Finite Element Computation against Analytical Result

Radical Distance	e9x2a	e9x2b	e9x2c	Analytical
0.000000e+00	8.719550e-02	8.454730e-02	8.333330e-02	8.333330e-02
1.625000e-01		8.161330e-02	8.113280e-02	8.113281e-02
3.250000e-01	7.545940e-02	7.476720e-02	7.453120e-02	7.453125e-02
4.625000e-01		6.565210e-02	6.550780e-02	6.550781e-02
6.000000e-01	5.363690e-02	5.340960e-02	5.333330e-02	5.333333e-02
7.125000e-01		4.107480e-02	4.102860e-02	4.102864e-02
8.250000e-01	2.669620e-02	2.663500e-02	2.661460e-02	2.661458e-02
9.125000e-01		1.395500e-02	1.394530e-02	1.394531e-02
1.000000e+00	1.338234e-11	6.111427e-12	1.440060e-12	0.000000e+00

Parameters, Options, and Subroutines Summary

Examples e9x2a.dat, e9x2b.dat, and e9x2c.dat:

Parameters	Model Definition Options
ELEMENTS	CONNECTIVITY
END	CONTINUE
FLUID	CONTROL
SIZING	COORDINATES
	DIST LOADS
	END OPTION
	FIXED VELOCITY
	ISOTROPIC
	POST
	PRINT ELEMENT
	STEADY STATE

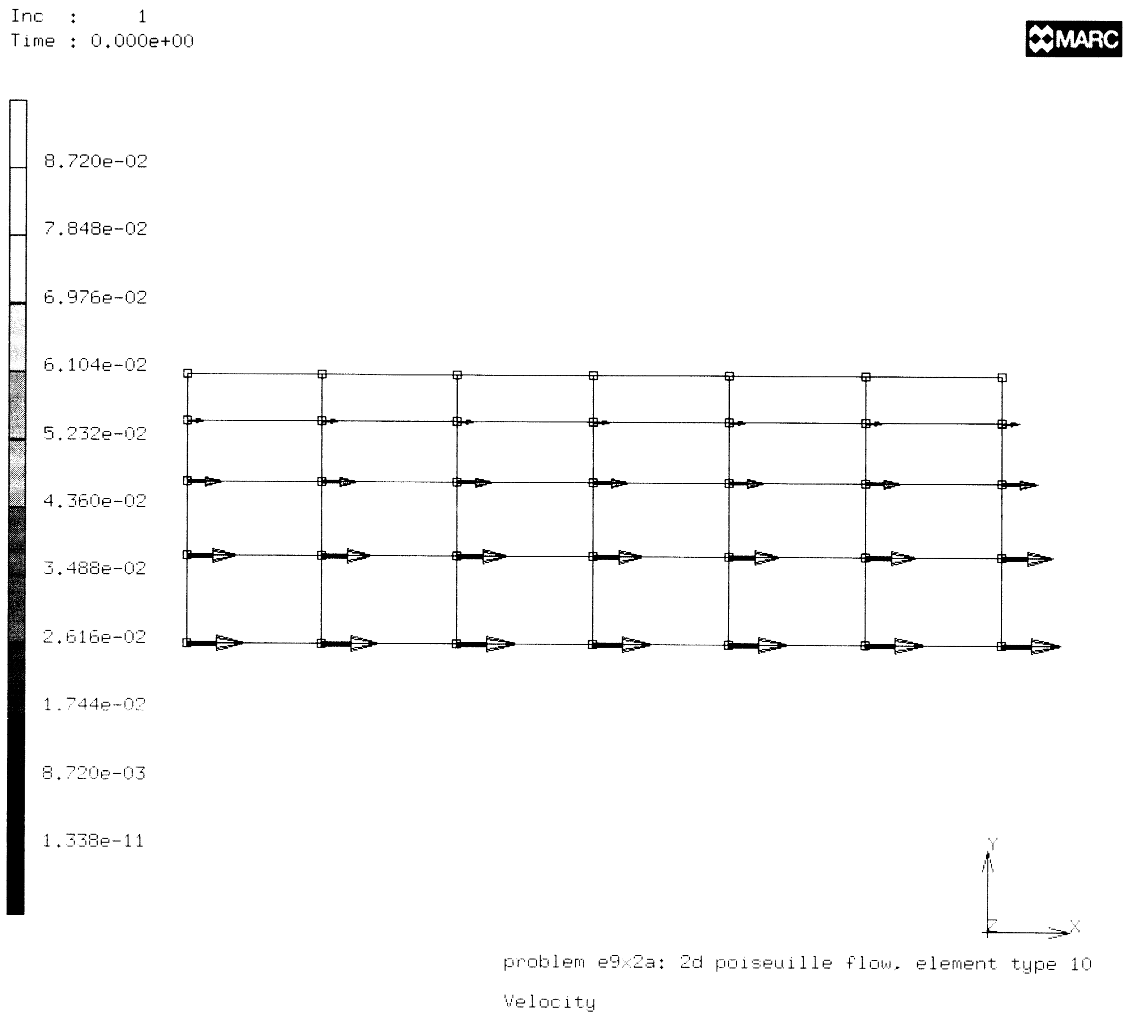


Figure 9.2-1 Vector Plot of the Poiseuille Flow Velocity Field, Discretized using Element Type 10 and the Mixed Method

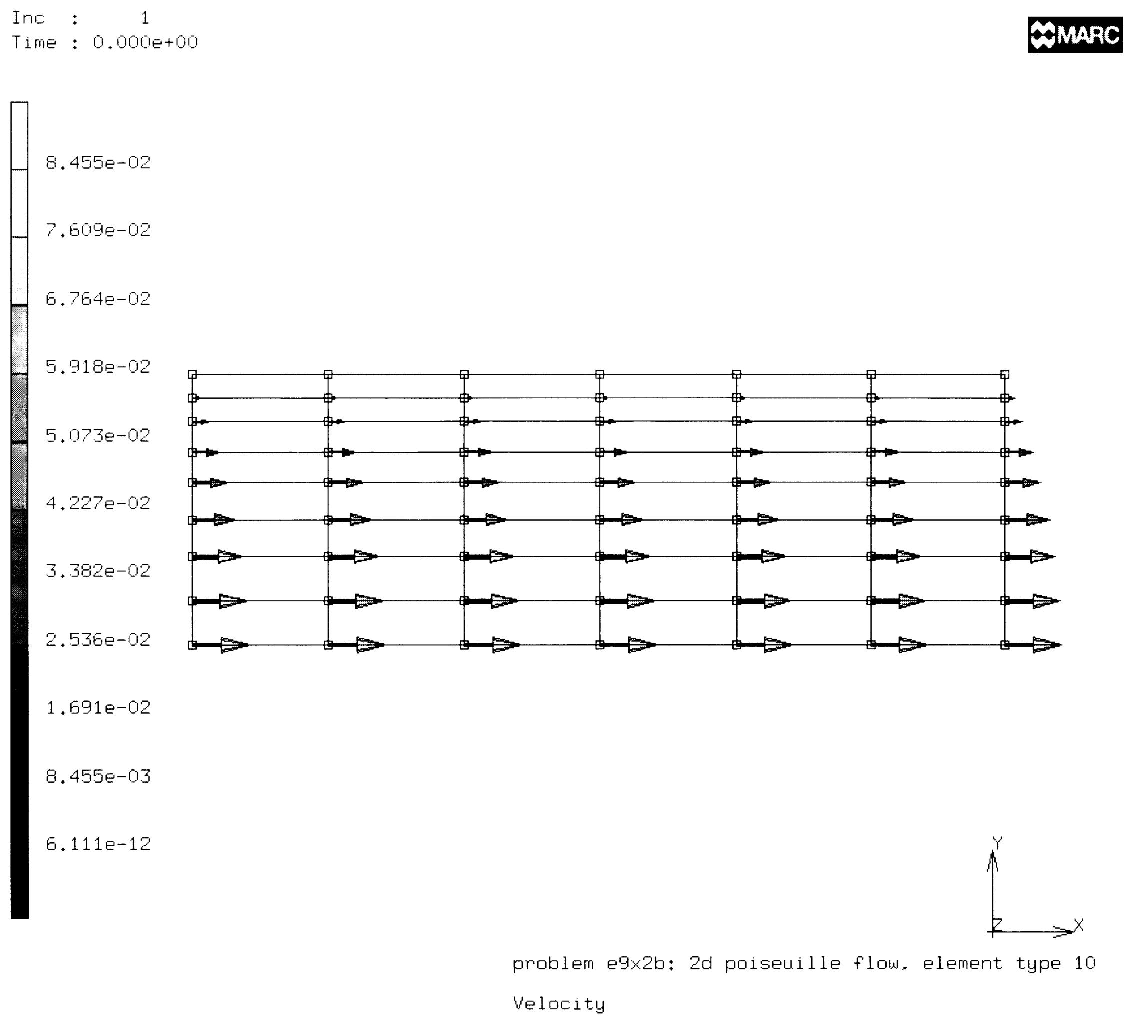


Figure 9.2-2 Vector Plot of the Poiseuille Flow Velocity Field, Discretized using Element Type 10 and the Mixed Method (Finer Mesh)

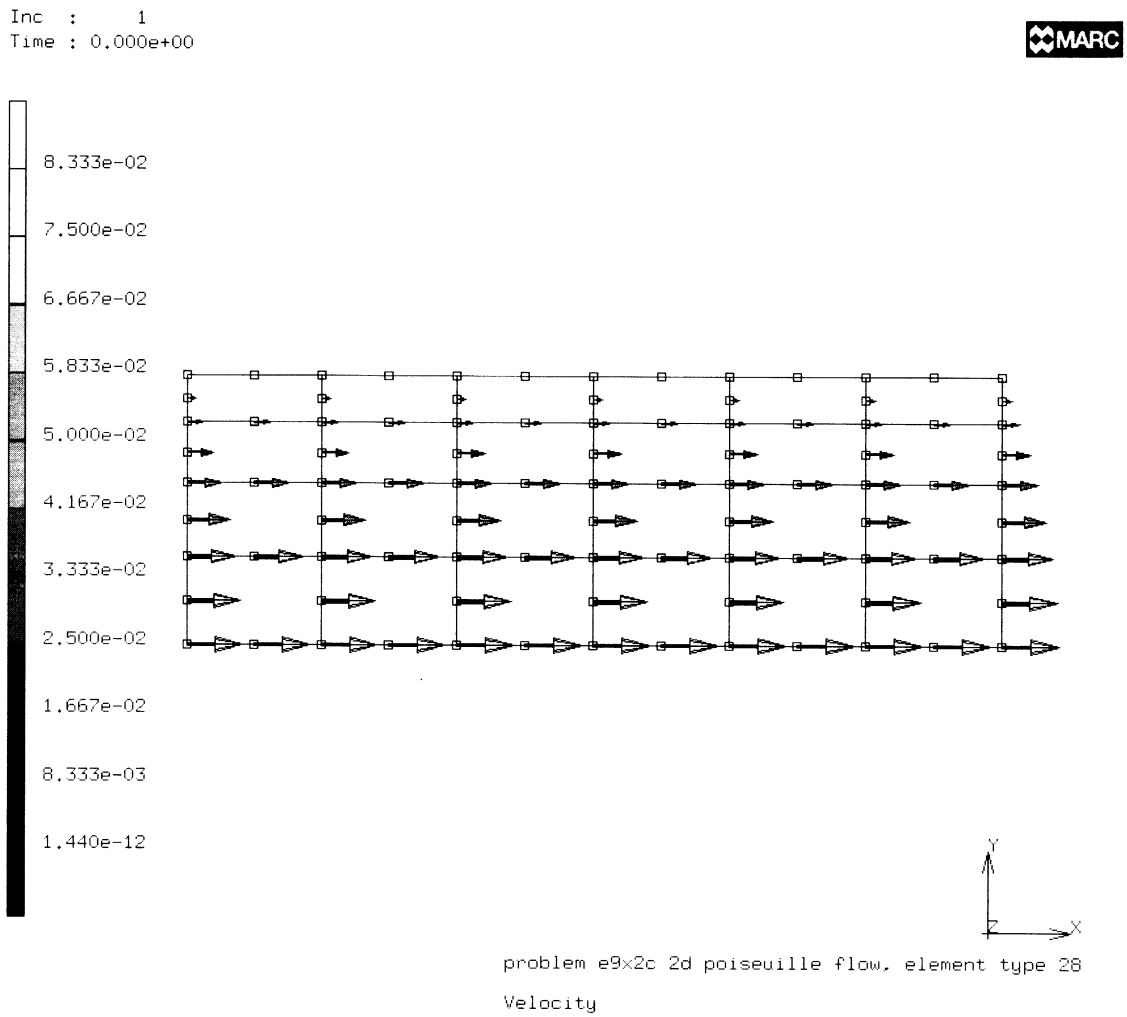


Figure 9.2-3 Vector Plot of the Poiseuille Flow Velocity Field, Discretized using Element Type 28 and the Mixed Method

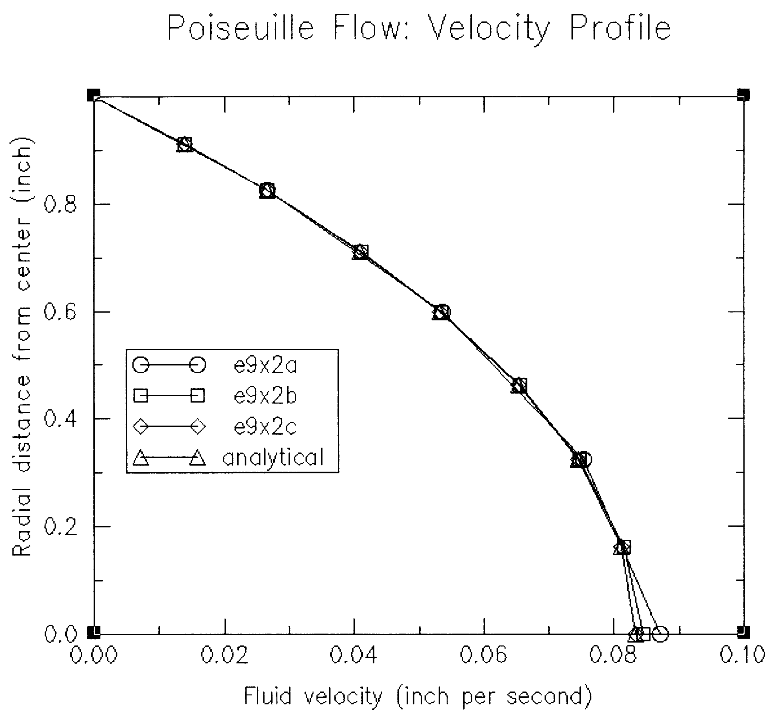


Figure 9.2-4 Comparison of Computation and Analytical Results

9.3 Fluid Squeezed Between Two Long Plates

This problem has an approximate analytical solution for viscous, incompressible fluids. The flow is generated by squeezing the fluid occupying the space in between two infinitely long, rigid plates. For this problem, the results obtained using steady state approximation, in combination with the mixed method, are presented.

This problem is modeled using the two techniques summarized below:

Data Set	Element Type(s)	Number of Elements	Number of Nodes
e9x3a	11	60	77
e9x3b	6	120	77

Element

Element type 11 is a lower-order, 4-node, bilinearly interpolated, planar element used in problem e9x3a to discretize the fluid domain. Three-noded triangular element of type 6 is used in problem e9x3b. Using the mixed method, each node has three degrees of freedom: two planar velocity components and a pressure.

Model

The six-by-two square inches of discretized fluid regions as shown in Figures 9.3-1 and 9.3-2 for problems e9x3a and e9x3b, respectively, represent a quadrant of the fluid domain obtained by considering symmetry with respect to both the x- and y-axis. The quadrilateral mesh has 60 elements, while the triangular mesh uses 120 elements. It is assumed here that the 1:3 aspect ratio chosen for the fluid domain is sufficient to accurately approximate the effects of infinitely long plates on the fluid.

Boundary Conditions

To model the squeezing effect from the top plate, the y component of fluid velocity along the top boundary is set to -1.0 inch per second; the x component is zero considering no-slip boundary condition. The left side of the domain is a symmetry line along the y-axis, so the velocity component along x-direction is set to zero. Also, the bottom side of the fluid domain is a symmetry line along the x-axis, hence the y-component of fluid velocity is given as zero.

Material

Newtonian fluid material with a viscosity of 1.0 lbf.sec/square inch and a mass density of 1.0E-06 pound per cubic inch is used to model this viscous flow.

Results

The vector plot of fluid velocity field is given in Figures 9.3-1 and 9.3-2, respectively, for problems e9x3a and e9x3b. The arrows representing velocity vectors are scaled according to their magnitues. The pressure distribution in the flow fields are given by the contour plots in Figures 9.3-3 and 9.3-4. Comparison of computation and analytical resut is given in Tables 9.3-1 and 9.3-2, and Figures 9.3-5 and 9.3-6.

Table 9.3-1 Comparison of Fluid Velocity X Obtained from Finite Element Computation against Analytical Result (at x=6)

Vertical Distance	e9x3a	e9x3b	Analytical
0.000000e+0	4.394480e+00	4.347870e+00	4.500000e+00
2.500000e-01	4.334840e+00	4.292710e+00	4.429688e+00
5.000000e-01	4.155030e+00	4.121650e+00	4.218750e+00
1.000000e+00	3.418850e+00	3.422630e+00	3.375000e+00
1.500000e+00	2.122560e+00	2.171330e+00	1.968750e+00
1.750000e+00	1.213830e+00	1.248630e+00	1.054688e+00
2.000000e+00	1.259060e-10	9.740150e-11	0.000000e+00

Table 9.3-2 Comparison of Fluid Velocity Y Obtained from Finite Element Computation against Analytical Result (at x=6)

Vertical Distance	e9x3a	e9x3b	Analytical
0.000000e+0	-3.249630e-11	-3.140290e-11	0.000000e+00
2.500000e-01	-1.570460e-01	-1.472210e-01	-1.865234e-01
5.000000e-01	-3.115110e-01	-2.918960e-01	-3.671875e-01
1.000000e+00	-6.053120e-01	-5.663410e-01	-6.875000e-01
1.500000e+00	-8.500980e-01	-8.076410e-01	-9.140625e-01
1.750000e+00	-9.561750e-01	-9.110980e-01	-9.775391e-01
2.000000e+00	-1.000000e+00	-1.000000e+00	-1.000000e+00

Parameters, Options, and Subroutines Summary

Examples e9x3a.dat and e9x3b.dat:

Parameters	Model Definition Options
ELEMENTS	CONNECTIVITY
END	CONTINUE
FLUID	CONTROL
SIZING	COORDINATES
	END OPTION
	FIXED VELOCITY
	ISOTROPIC
	POST
	PRINT ELEMENT
	STEADY STATE

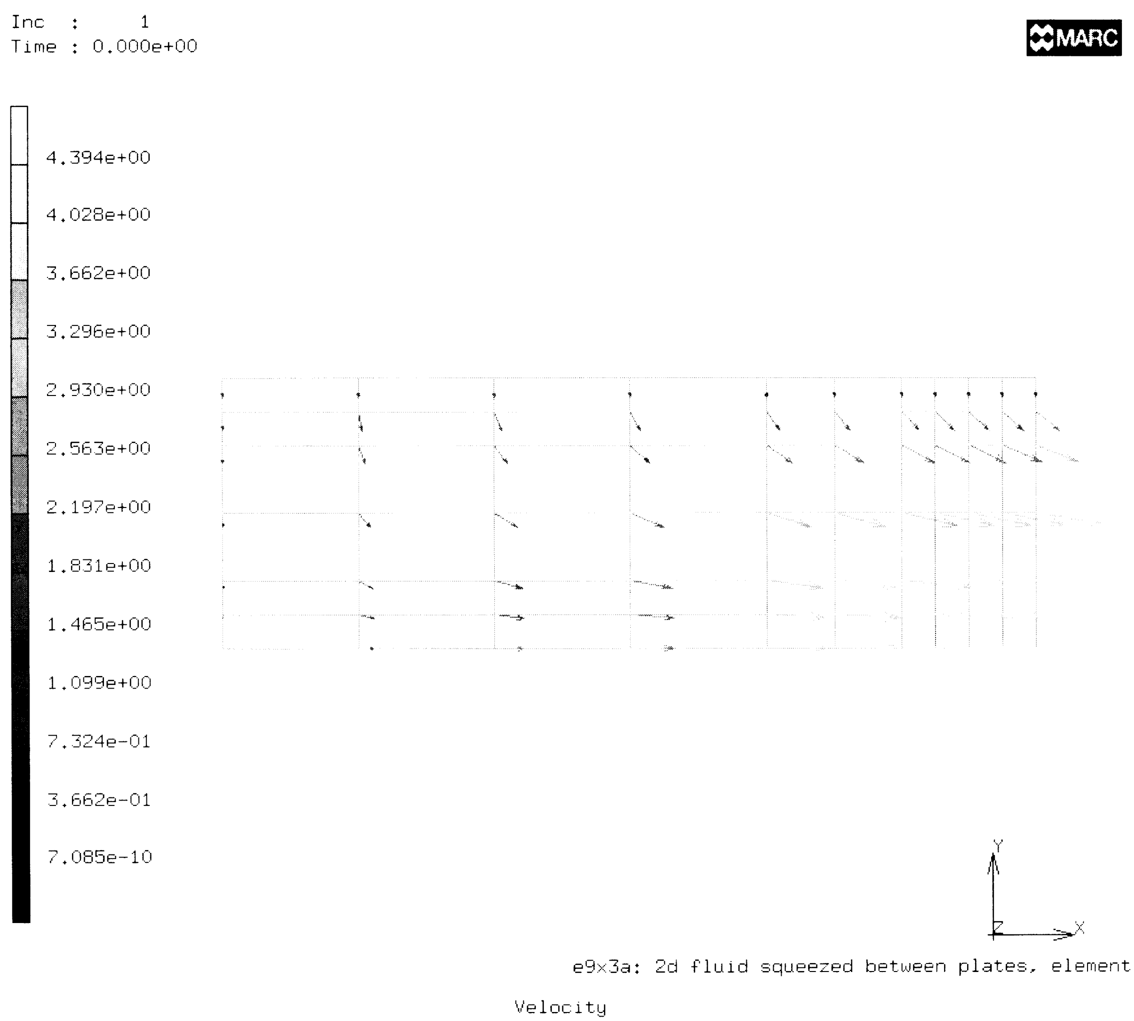


Figure 9.3-1 Vector Plot of the Squeezed Fluid Velocity Field, Discretized using Element Type 11 and the Mixed Method

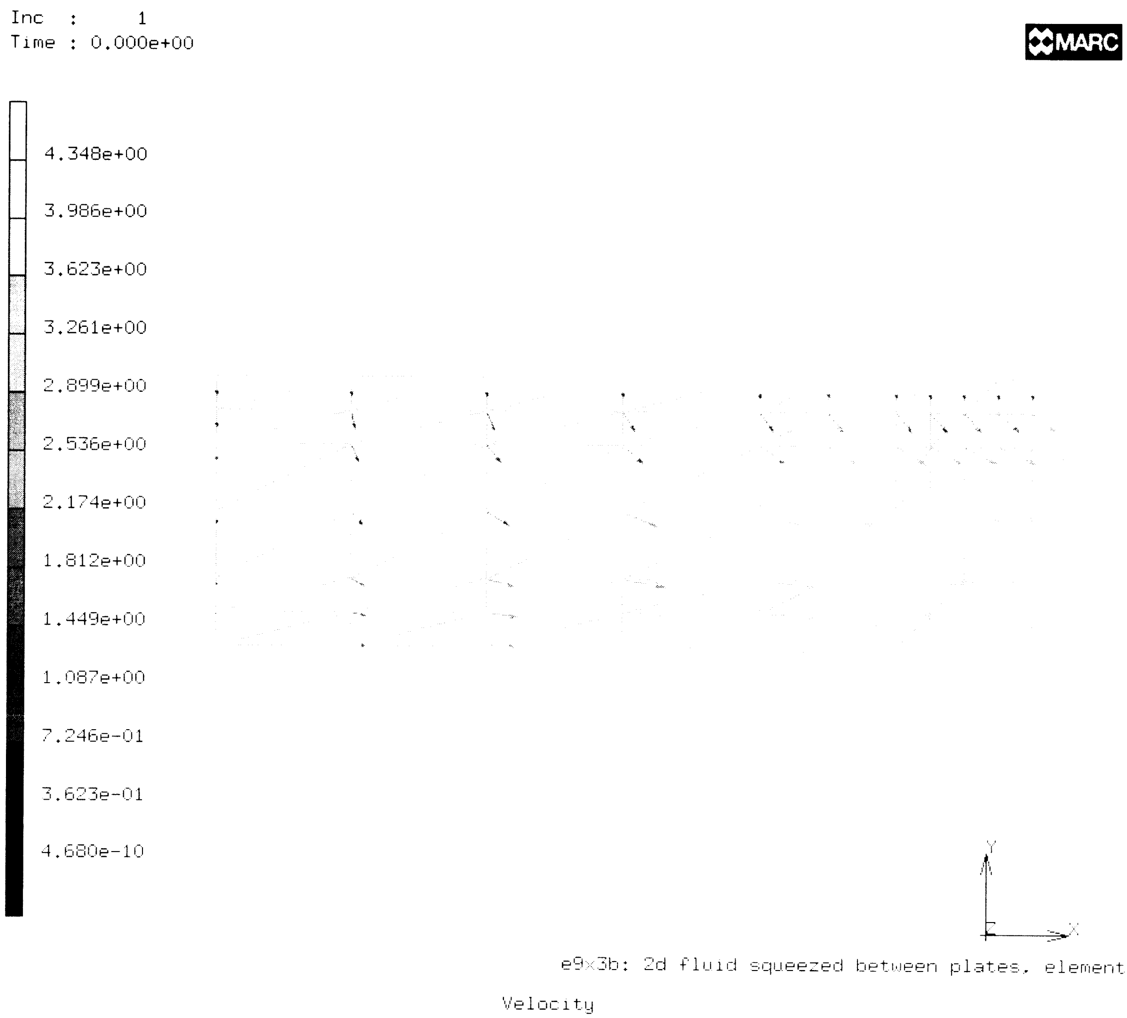


Figure 9.3-2 Vector Plot of the Squeezed Fluid Velocity Field, Discretized using Element Type 6 and the Mixed Method

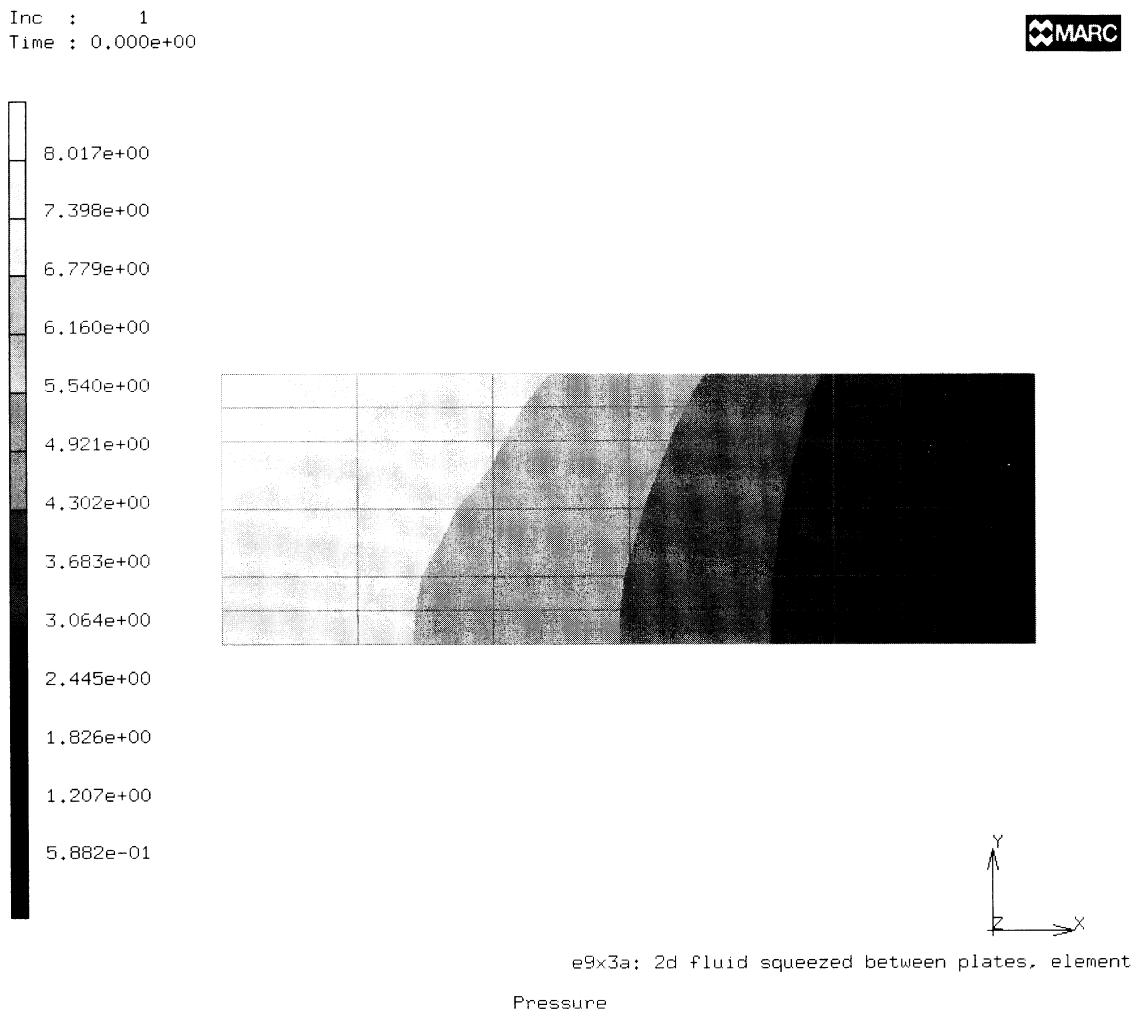


Figure 9.3-3 Contour Plot of the Squeezed Fluid Pressure Field, Discretized using Element Type 11 and the Mixed Method

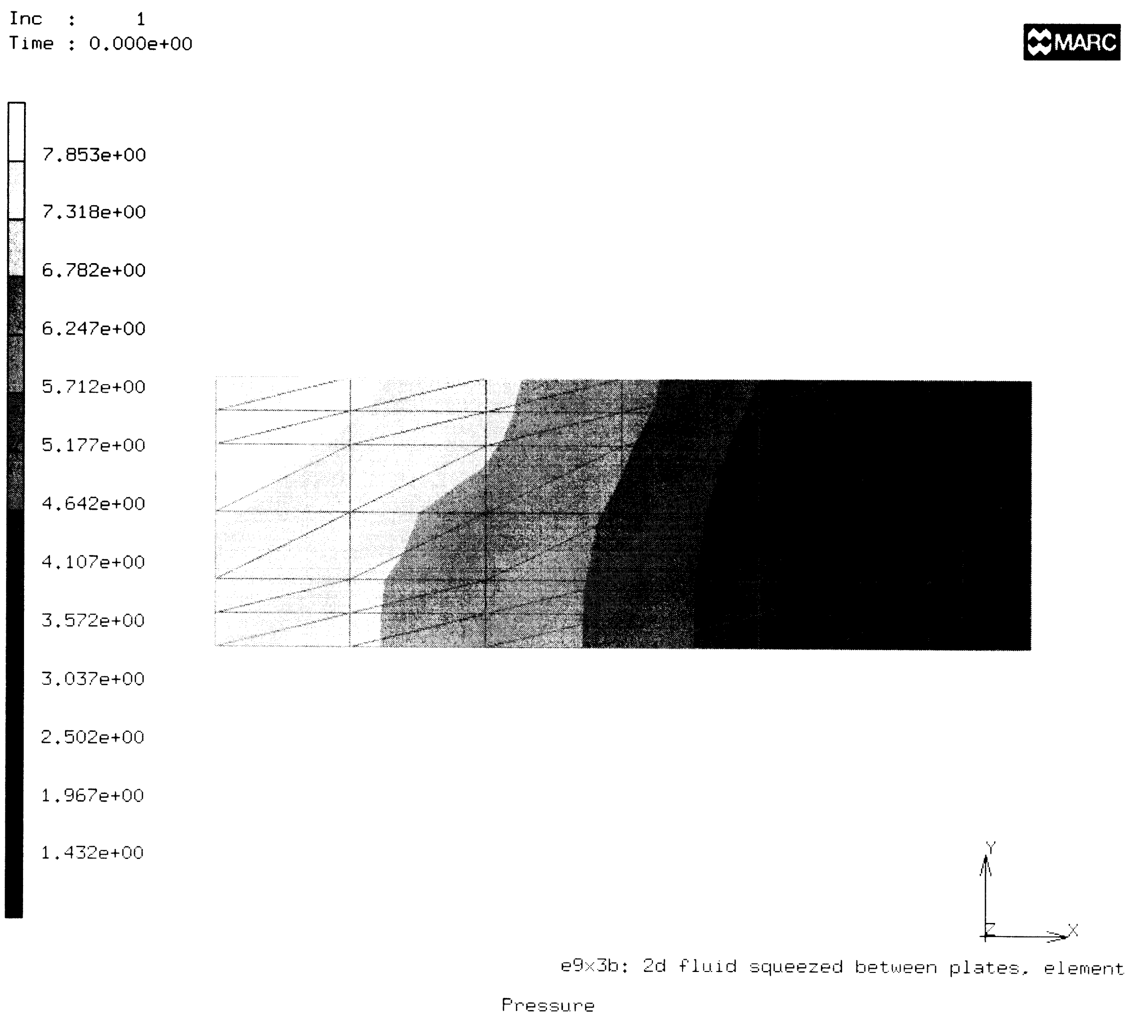


Figure 9.3-4 Contour Plot of the Squeezed Fluid Pressure Field, Discretized using Element Type 6 and the Mixed Method

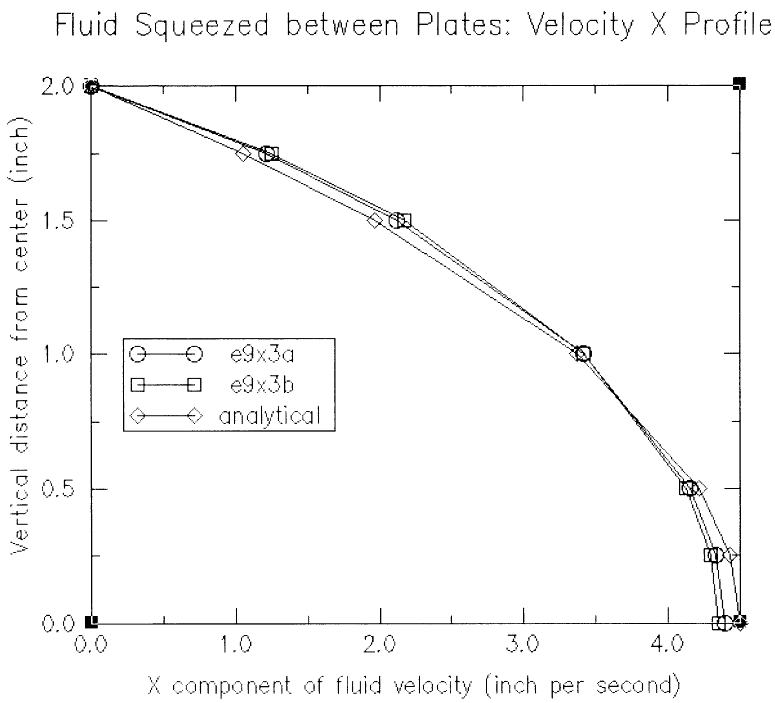


Figure 9.3-5 Comparison of Fluid Velocity V_x Computation and Analytical Results at x=6

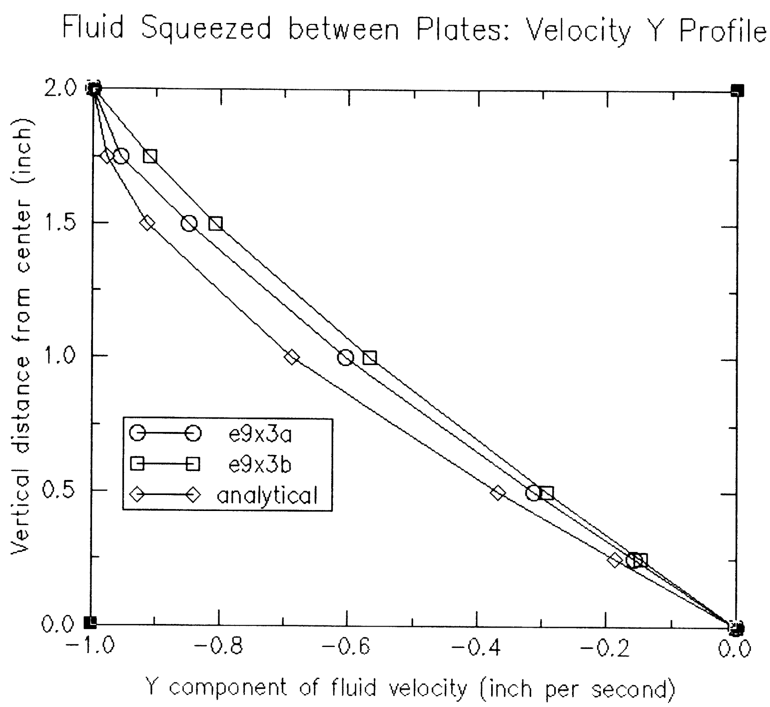


Figure 9.3-6 Comparison of Fluid Velocity V_y Computation and Analytical Results at x=6

9.4 Driven Cavity Flow

This is a commonly used problem to demonstrate a modeling of viscous, incompressible flow using Navier-Stokes equations. The flow is driven by a rigid lid sliding on top of the fluid filled cavity, which creates circulating flow in the fluid. For this problem, the result is obtained using steady state approximation in combination with the mixed method. Using the mixed method, Streamline Upwind Petrov-Galerkin (SUPG) and Pressure Stabilizing Petrov-Galerkin (PSPG) techniques are automatically invoked by MARC to prevent numerical instability, which is frequently observed as nonphysical wiggles in the flow field.

Element

Element type 11 is a lower-order, 4-node, bilinearly interpolated, planar element used in the problem to discretize the fluid domain. Using the mixed method, each node has three degrees of freedom: two planar velocity components and a pressure.

Model

The six-by-six square inches domain of the fluid is uniformly meshed as shown in Figure 9.4-1 using a total of 144 elements with 12-element discretization on each side.

Boundary Conditions

The fluid filled cavity is confined by rigid walls on its three sides; that is, left, right, and bottom. No-slip boundary condition requires that both components of the fluid velocity along the walls be set to zero. Flow in the cavity is driven by a rigid lid on top of the cavity, moving with a velocity of 1.0 inch per second along the negative x-direction. As such, the x components of the nodal velocities along the top of the cavity are assigned a value of -1.0 inch per second, and the corresponding y components are set to zero.

Material

Newtonian fluid material with a viscosity of 1.0 lbf.sec/square inch and a mass density of 1.0 pound per cubic inch is used to model this highly viscous flow. Reynolds number for this problem is less than 1.

Results

The vector plot of fluid velocity field is given in Figure 9.4-1 where the arrows are scaled according to their magnitudes. The circulating flow field as a result of the moving lid is shown. The pressure distribution in the flow field is given by the contour plot in Figure 9.4-2.

Parameters, Options, and Subroutines Summary

Example e9x4.dat:

Parameters	Model Definition Options
ELEMENTS	CONNECTIVITY
END	CONTINUE
FLUID	CONTROL
SIZING	COORDINATES
	END OPTION
	FIXED VELOCITY
	PRINT ELEMENT
	ISOTROPIC
	POST
	STEADY STATE

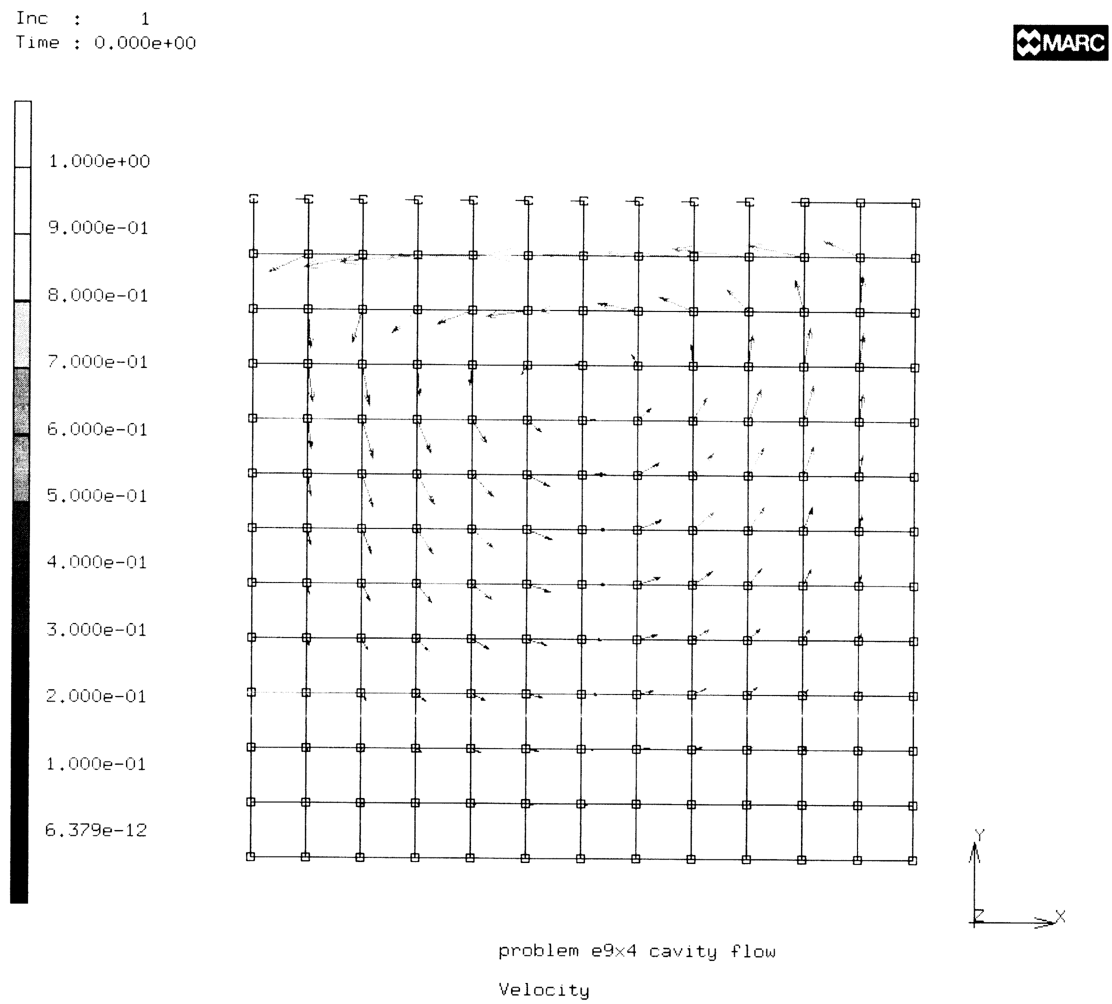


Figure 9.4-1 Vector Plot of the Driven Cavity Flow Velocity Field, Discretized using Element Type 11 and the Mixed Method

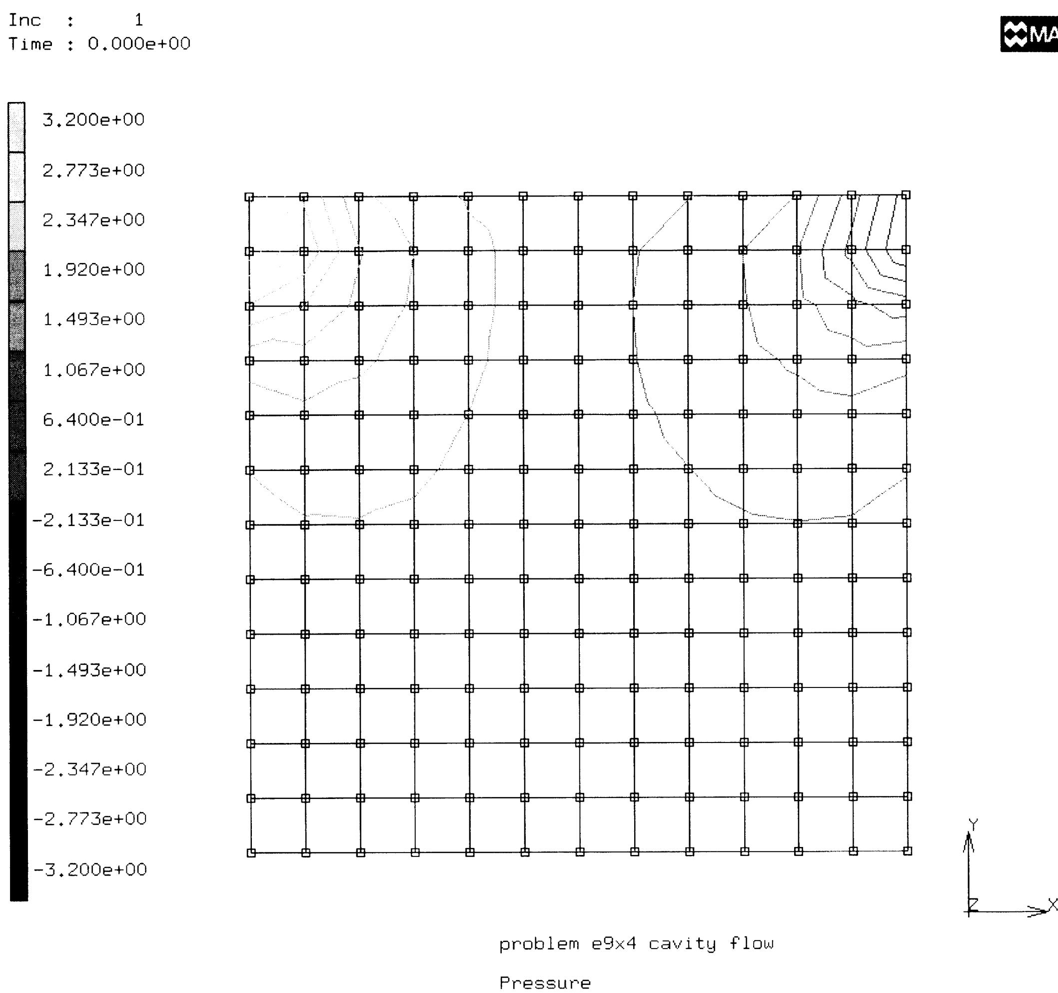


Figure 9.4-2 Contour Plot of the Driven Cavity Flow Pressure Field, Discretized using Element Type 11 and the Mixed Method

9.5 Flow Past a Circular Cylinder

This problem simulates the flow past a cylinder. This problem is performed as both a steady state analysis and transient analysis based on Navier-Stokes equations.

This problem is modeled using the five techniques summarized below.

Data Set	Element Type(s)	Number of Elements	Number of Nodes	Differentiating Features
e9x5a	11	440	483	Mixed method, steady state
e9x5b	11	440	483	Penalty method, steady state
e9x5c	27	440	1405	Mixed method, steady state
e9x5d	27	440	1405	Penalty method, steady state
e9x5e	11	440	483	Mixed method, transient method

Element

Element type 11 is a 4-node planar element using bilinear interpolation functions. Element type 27 is an 8-node planar element using biquadratic interpolation functions.

Model

A planar model of the flow is simulated. Because of symmetry conditions, only one half of the model is meshed. The cylinder has a radius of 1 inch. The channel is given a length of 5 inches in the upstream direction and a length of 10 inches in the downstream direction. The model is given a depth of 10 inches with the desire that this is enough to accurately capture the fluid behavior. The finite element mesh, consisting of 4-node elements, is shown in Figure 9.5-1. The 8-node element mesh is shown in Figure 9.5-2.

Boundary Conditions

Along the symmetry axis (Y = 0) $V_y = 0$. Along the upstream boundary condition, steady state fluid conditions are considered with $V_x = 1.0$ and $V_y = 0$ for problems e9x5a through e9x5e. Along the outlet downstream boundary, the fluid is considered traction free. At y = 5.0, the velocity is $V_x = 1$, $V_y = 0$.

Material

The fluid is treated as Newtonian with a viscosity of 1.0 lbf.sec/square inch and a mass density of 1.0 lb/cubic inch.

Numerical Procedure

In all of the analyses, the Newton Rapshon procedure is used to solve the nonlinear problem. In the transient analysis, a fixed time step procedure is used. Convergence is based upon the relative velocity criteria.

Results

The fluid flow has three different behaviors based upon the axial position. In the upstream area, the fluid flow is virtually parallel. In the region near the cylinder, three behaviors are observed. First, the fluid is deflected along the cylinder. Second, near the body, a boundary layer develops where the viscous behavior dominates. Third, as the cylinder acts to constrict the flow, the velocity in the region at X = 5 increases to satisfy incompressibility. At steady state, the Reynolds number is about 100.

Figures 9.5-4 and 9.5-5 show the pressure distribution in the fluid for problems e9x5a and e9x5c, respectively. Figure 9.5-6 shows the results for the transient analysis at the tenth increment. Figures 9.5-1 through 9.5-3 show the vector plots of the velocity for the analysis of problems e9x5a, e9x5c, and 39x5e, respectively.

Parameters, Options, and Subroutines Summary

Examples e9x5a.dat, e9x5b.dat, e9x5c.dat, and e9x5d.dat:

Parameters	Model Definition Options
ELEMENTS	CONNECTIVITY
END	COORDINATES
FLUID	CONTINUE
SIZING	CONTROL
	END OPTION
	FIXED VELOCITY
	ISOTROPIC
	POST
	PRINT ELEMENT
	STEADY STATE

Examples e9x5e.dat:

Parameters	Model Definition Options	History Definition Options
ELEMENTS	CONNECTIVITY	CONTINUE
END	CONTROL	TRANSIENT NON AUTO
FLUID	COORDINATES	
SIZING	END OPTION	
	FIXED VELOCITY	
	INITIAL VEL	
	ISOTROPIC	
	POST	
	PRINT ELEMENT	

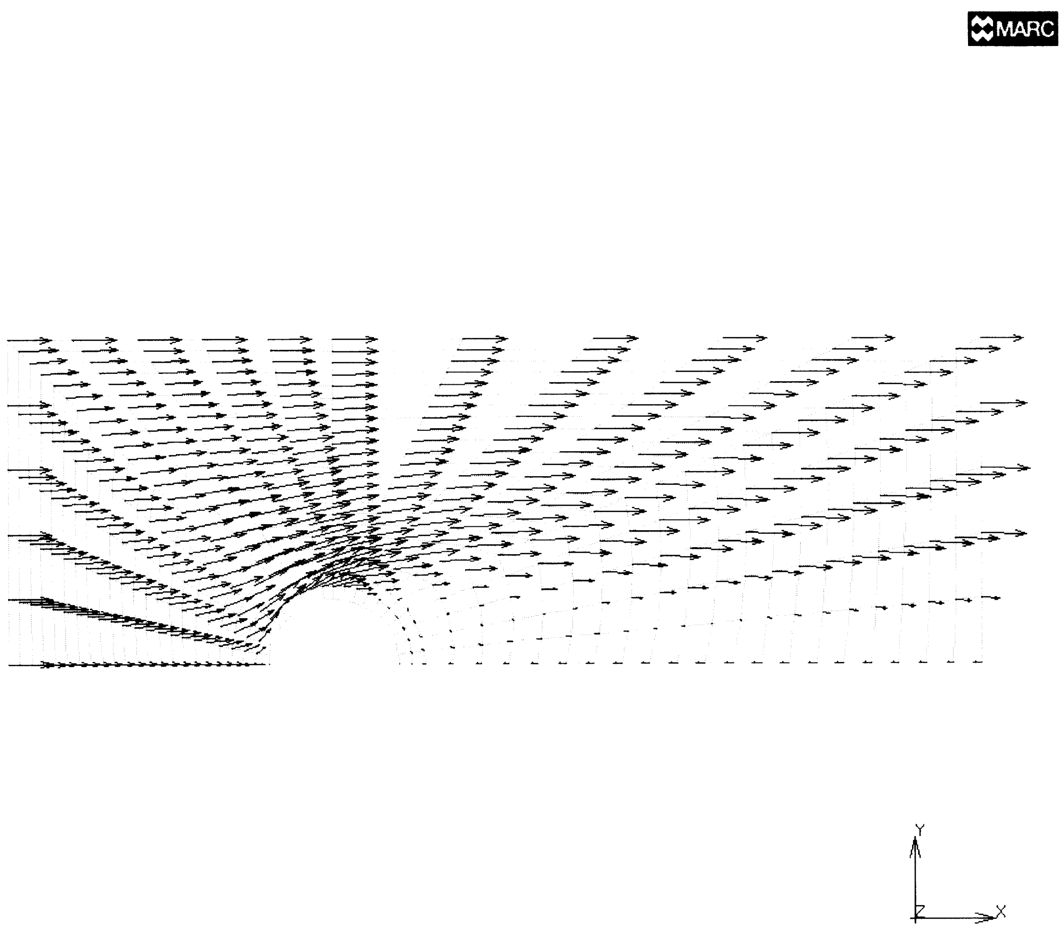


Figure 9.5-1 Vector Plot of the Flow Over Cylinder Velocity Field, Discretized using Element Type 11 and the Mixed Method

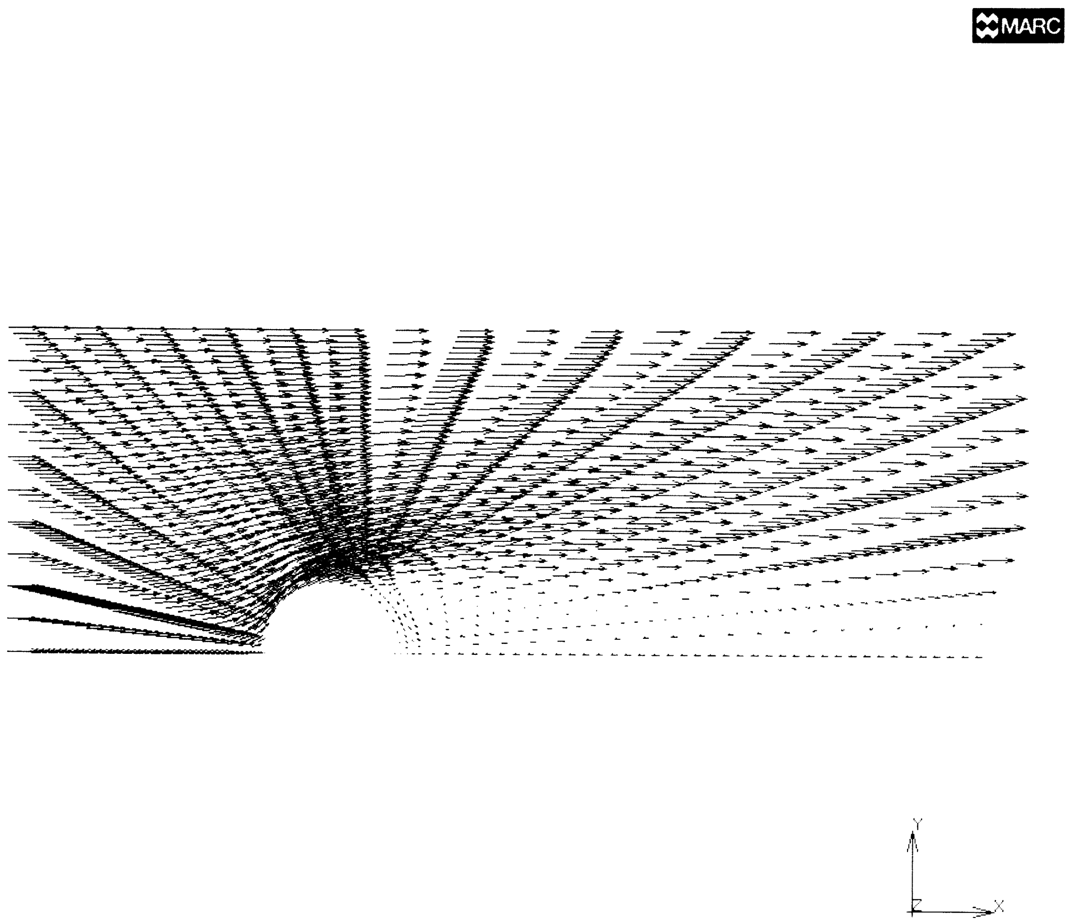


Figure 9.5-2 Vector Plot of the Flow Over Cylinder Velocity Field, Discretized using Element Type 27 and the Mixed Method

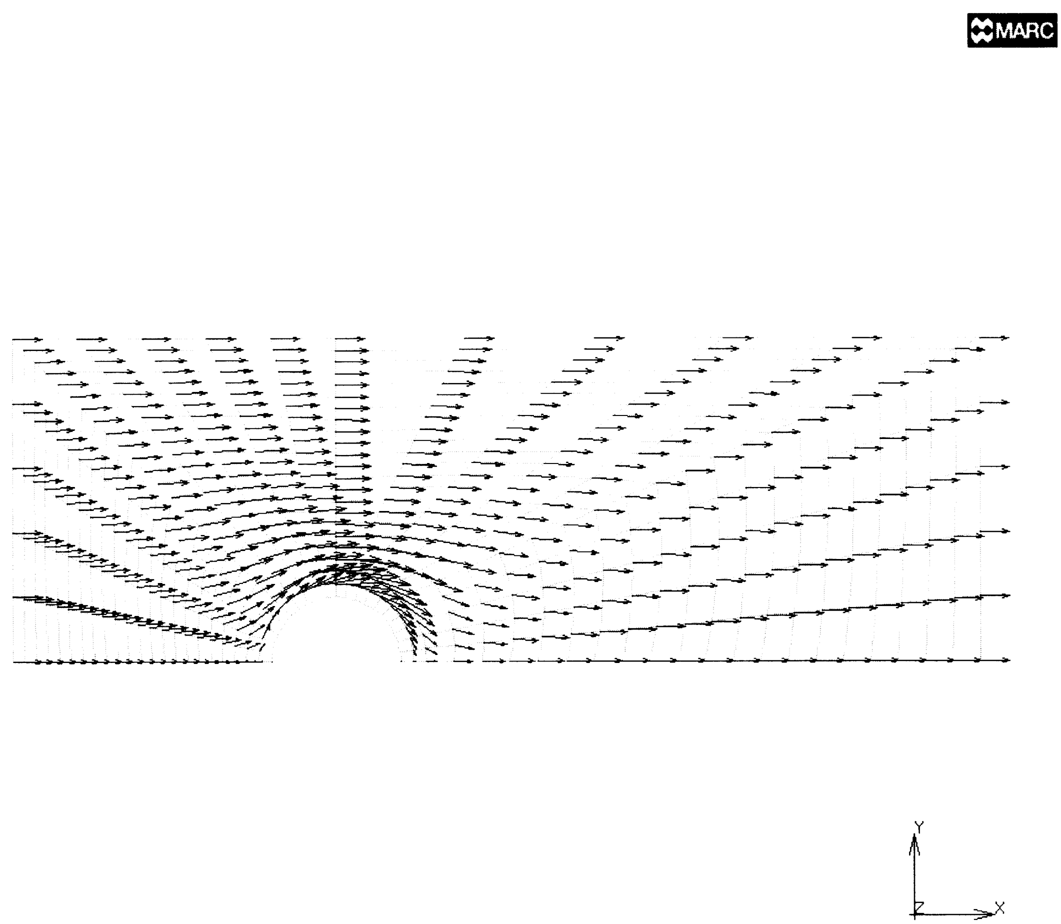


Figure 9.5-3 Vector Plot of the Flow Over Cylinder Transient Velocity Field at the Tenth Increment, Discretized using Element Type 11 and the Mixed Method

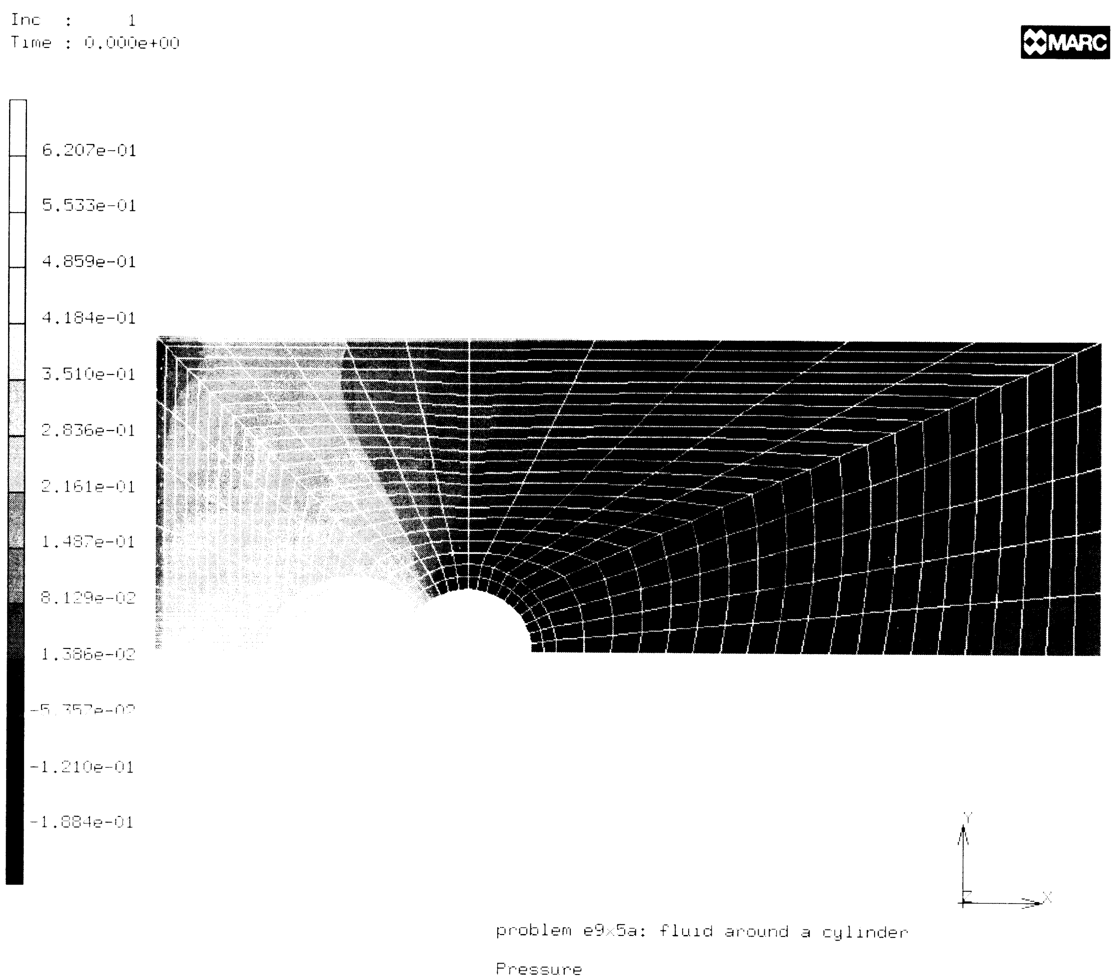


Figure 9.5-4 Contour Plot of the Flow Over Cylinder Pressure Field, Discretized using Element Type 11 and the Mixed Method

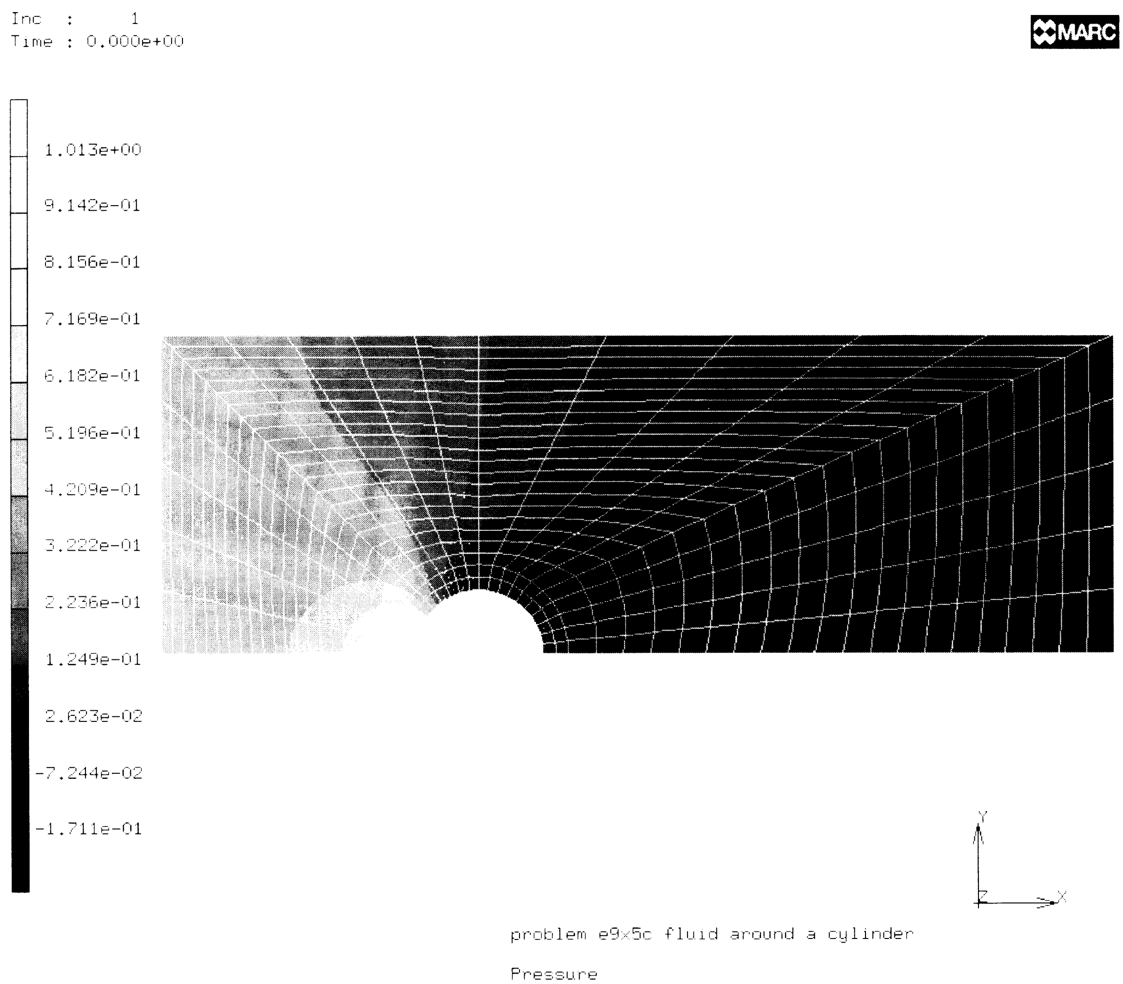


Figure 9.5-5 Contour Plot of the Flow Over Cylinder Pressure Field, Discretized using Element Type 27 and the Mixed Method

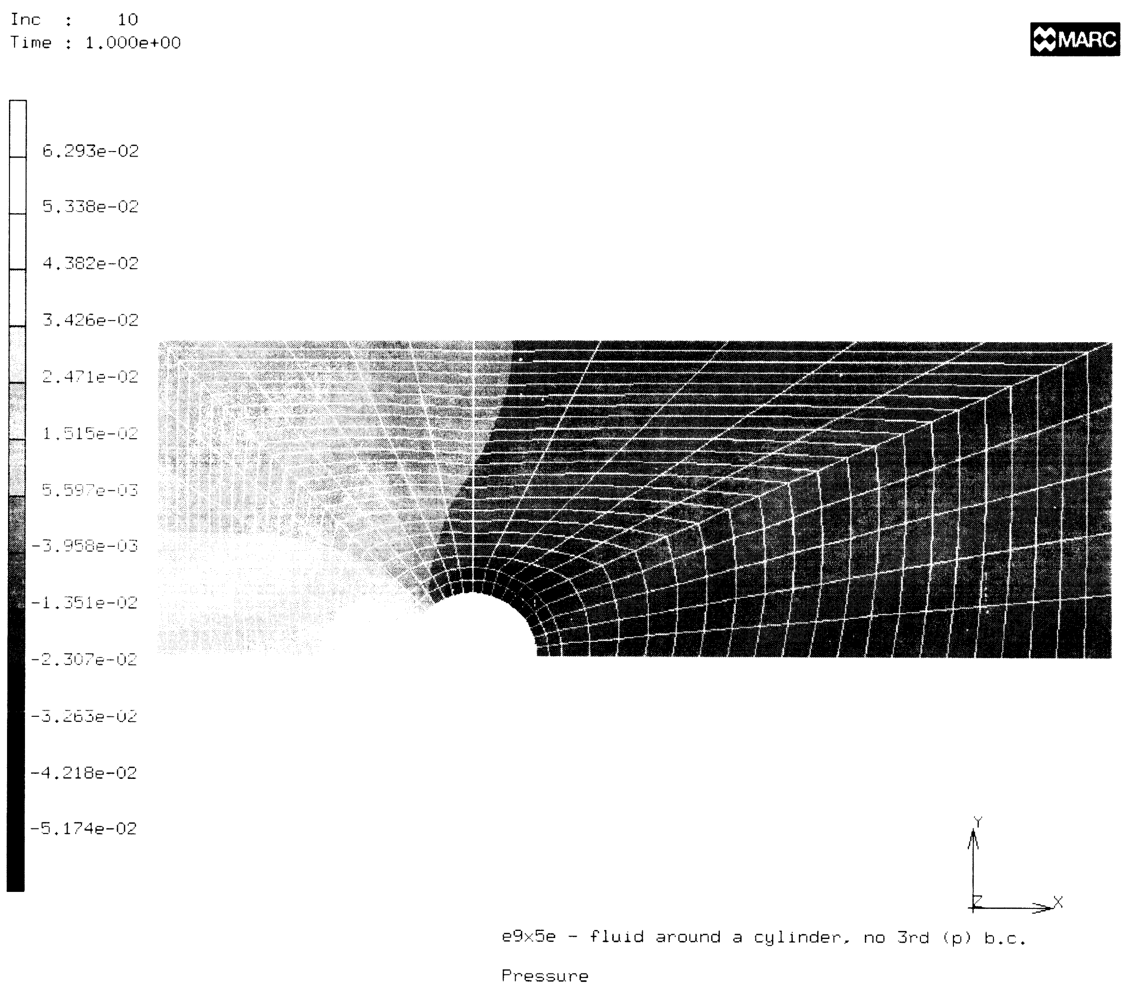


Figure 9.5-6 Contour Plot of the Flow Over Cylinder Transient Pressure Field at the Tenth Increment, Discretized using Element Type 11 and the Mixed Method

9.6 Flow Over Electronic Chip

This problem demonstrates coupled fluid-thermal behavior for the flow about electronic circuit chips. The flow is treated as two-dimensional and, in this example, the chips are considered to be at a constant temperature.

This problem is modeled using the two techniques summarized below.

Data Set	Element Type(s)	Number of Elements	Number of Nodes	Differentiating Features
e9x6a	11	311	370	Mixed method
e9x6b	11	311	370	Penalty method

Element

Element type 11, a 4-node isoparametric element, is used. For problem e9x6a, the mixed formulation procedure is used so the degrees of freedom are the velocities V_x, V_y, pressure, and the temperature. Problem e9x6b uses the penalty method which does not explicitly represent pressure as a nodal variable.

Model

The model is shown in Figure 9.6-1. The height of the channel is 1 inch and the chips have a dimension of 0.4 x 0.4 square inch and are separated by 1.0 inch. The amount of separation between the chips is significant as either a wake or recirculating flow can occur, depending on both the distance and the inlet velocity. The domain is discretized using 311 elements.

Boundary Conditions

The fluid enters the region at x = 0 with a velocity of $V_x = 1.0$ inch/second and $V_y = 0$. At the outflow section on the perimeter, the y-component of fluid velocity is set to zero. Other than the inlet and outlet sections, all velocity components are set to zero due to no-slip boundary conditions. Temperature along the perimeter of the domain are set to zero, except those nodes along the chips, which are set to 1°F.

Material

Newtonian fluid with a viscosity of 1.0 lbf.sec/square inch and a mass density of 1.0 lb/cubic inch is used to model the viscous flow. Thermal conductivity of the fluid is given as 0.0145 Btu/sec/in/°F.

Results

The vector plot of fluid velocity field for problem e9x6a is given in Figure 9.6-1 where the arrows are scaled according to their magnitude. The contour plots of temperature and pressure distributions are shown in Figures 9.6-2 and 9.6-3, respectively. Results obtained for problem e9.6b are indistinguishable from the above; therefore, they are not presented here.

Parameters, Options, and Subroutines Summary

Example e9x6a.dat and e9x6b.dat:

Parameters	Model Definition Options
ELEMENTS	CONNECTIVITY
END	CONTINUE
FLUID	CONTROL
SIZING	COORDINATES
	END OPTION
	FIXED VELOCITY
	ISOTROPIC
	POST
	PRINT ELEMENT
	STEADY STATE

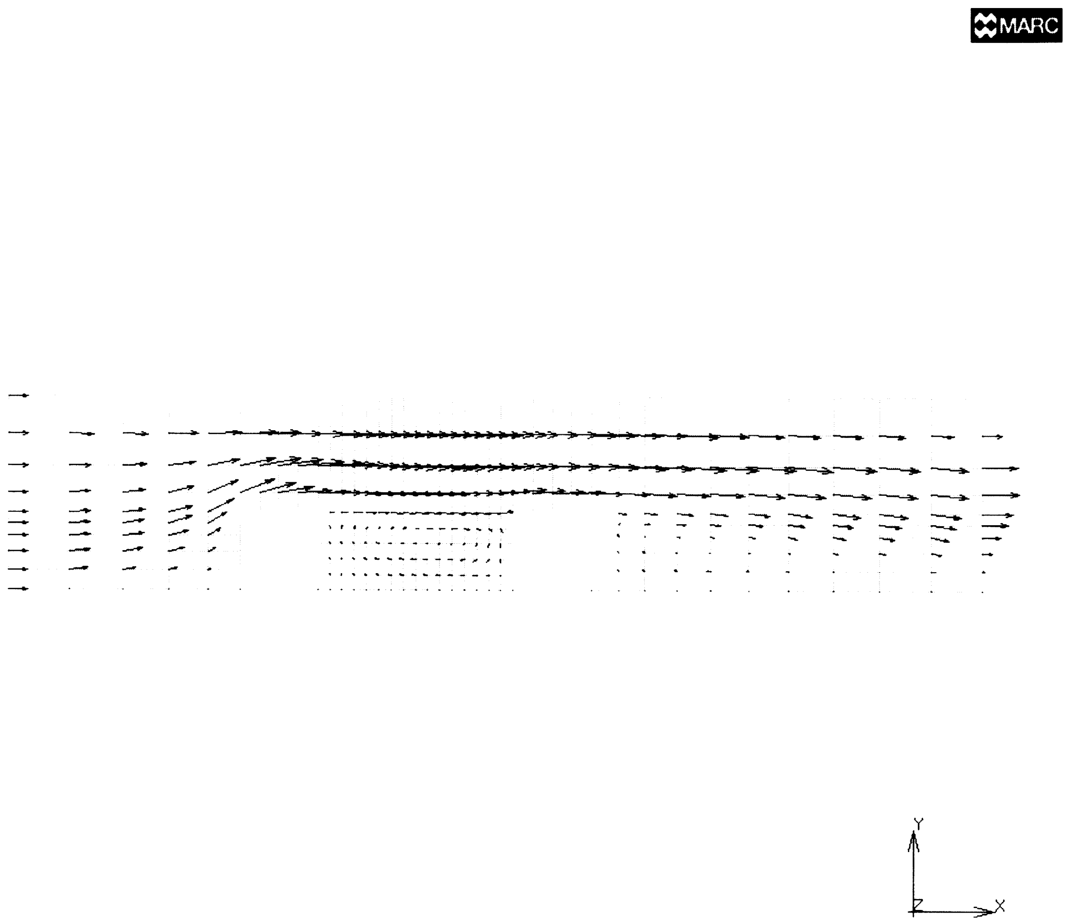


Figure 9.6-1 Vector Plot of the Flow Over Multiple Steps Velocity Field, Discretized using Element Type 11 and the Mixed Method

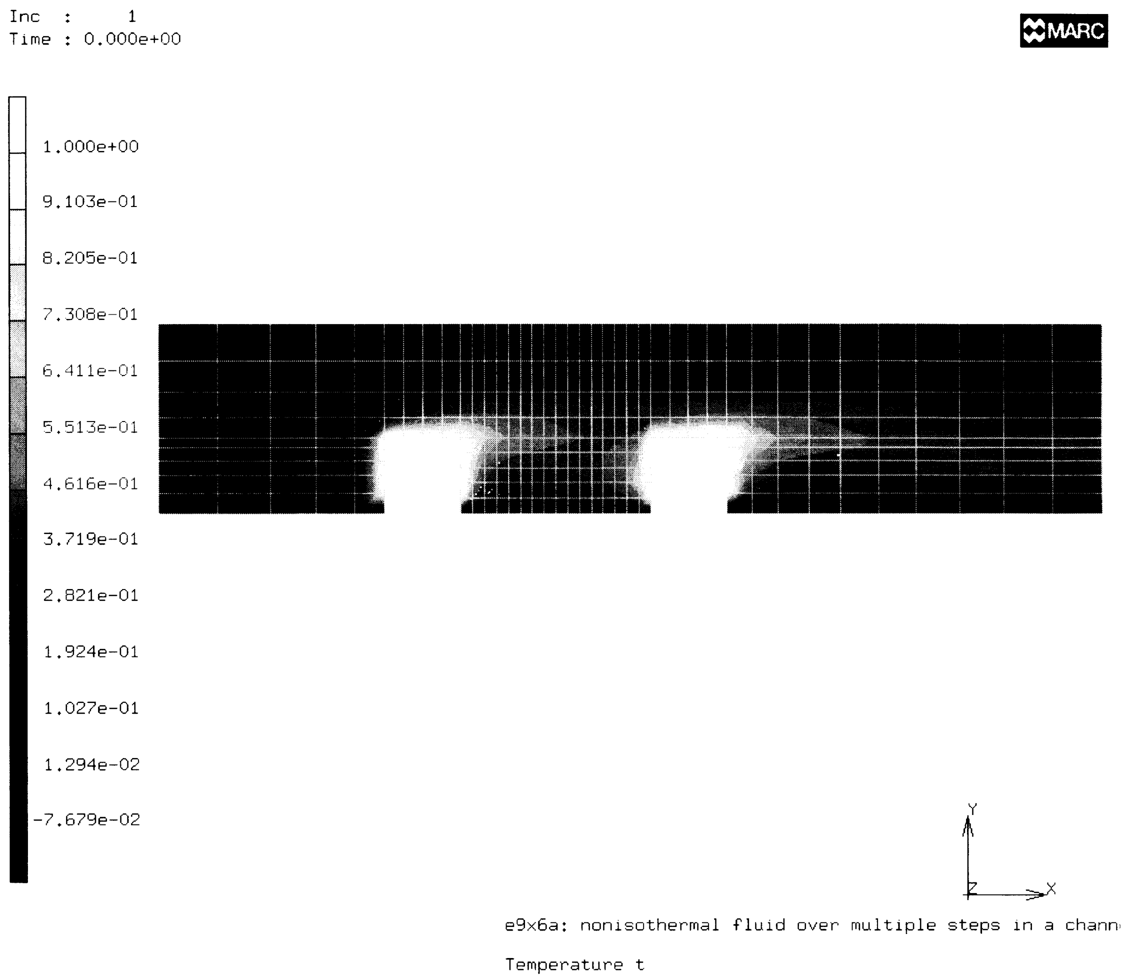


Figure 9.6-2 Contour Plot of the Flow Over Multiple Steps Temperature Field, Discretized using Element Type 11 and the Mixed Method

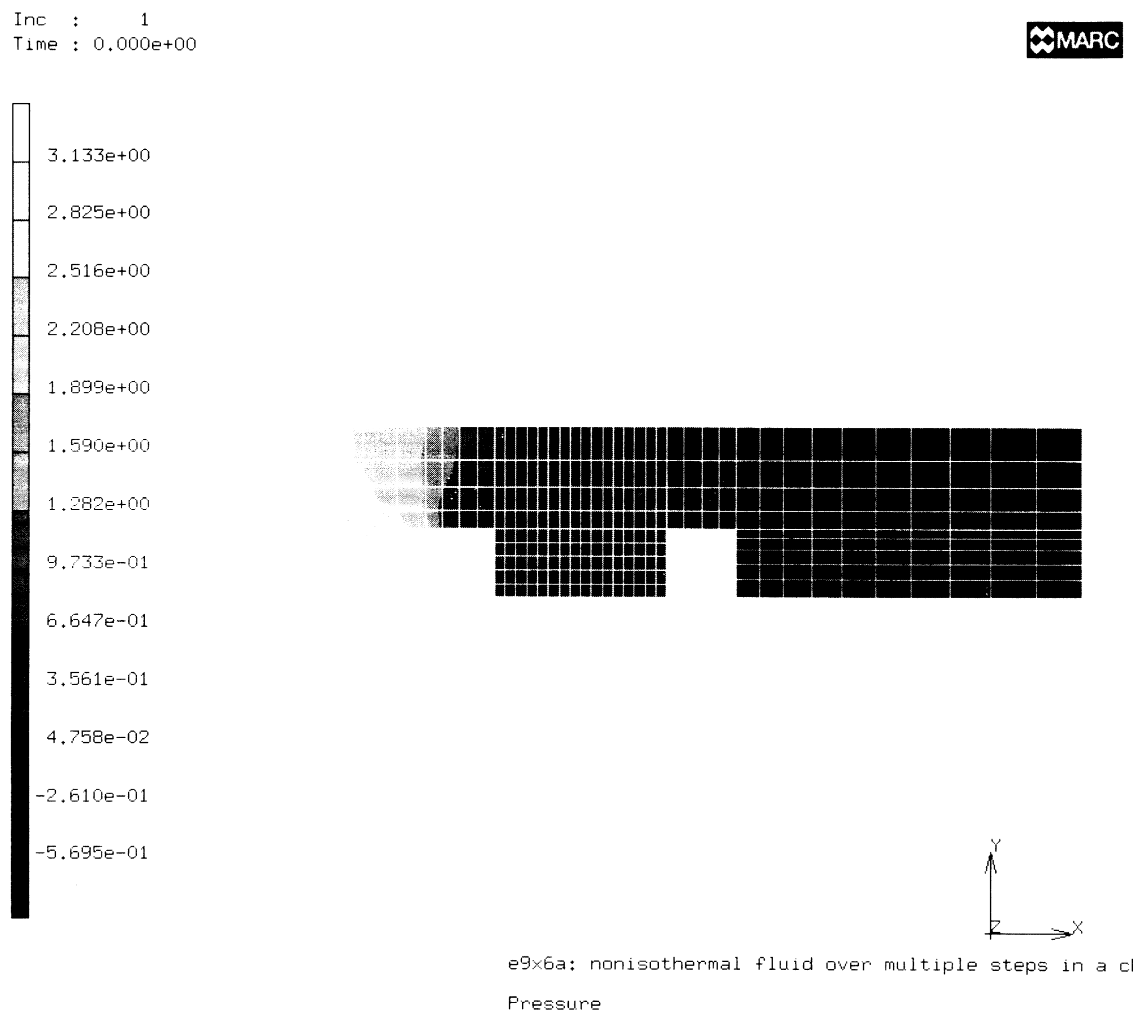


Figure 9.6-3 Contour Plot of the Flow Over Multiple Steps Pressure Field, Discretized using Element Type 11 and the Mixed Method

9.7 Natural Convection

A set of problems showing thermally induced fluid circulation in a cavity is presented to demonstrate buoyancy-driven, natural convection phenomena. This analysis feature is suitable for applications in electronic packaging and solidification process of metal castings, among others. Basically, the problems invoke coupling of heat transfer and fluid mechanics by way of density variation due to nonuniform temperature distribution. Boussinesq approximation is used by MARC to model this type of convective flows. Different levels of Rayleigh numbers are used in the following two cases to demonstrate MARC analysis capability.

This problem is modeled using the two techniques summarized below.

Data Set	Element Type(s)	Number of Elements	Number of Nodes	Differentiating Features
e9x7a	11	144	169	Coarser mesh
e9x7b	11	196	225	Finer mesh

Element

Element type 11 is a lower-order, 4-node, bilinearly interpolated, planar element used in this problem to discretize the fluid medium. Using the penalty method for fluid elements, and including the coupling with heat transfer, each node ends up with three degrees of freedom: two planar velocity components and a temperature.

Model

The one-by-one square inch domain of fluid is meshed as shown in Figures 9.7-1 and 9.7-2 for problems e9x7a and e9x7b, respectively. Problem e9x7a uses a total of 144 elements, with twelve-element discretization per side. On the other hand, problem e9x7b uses 14 element discretization per side, which results in a total of 196 elements. Graded meshes are used in both problems, with finer elements positioned closer to the perimeter of the fluid filled cavity to capture the steeper velocity gradient.

Load and Boundary Conditions

Gravity field oriented in the negative y direction is specified using load type 102. This is necessary in order to model buoyancy effects. The magnitude of gravity acceleration in this case is given by 1.0 force per unit mass. The fluid filled cavity is confined by rigid walls on all four sides. No-slip boundary conditions require that both components of fluid velocity along the walls be set to zero. Flow in the cavity is induced by the temperature difference between the left-side and right-side walls. For both cases, the temperature of the left-side wall is set at 2.0°F., which is also the reference temperature for the problems. The temperature of the right-side wall is given at 3.0°F for both problems.

Material

Newtonian fluid material with a viscosity of 1.0 lbf.sec/square inch and a mass density of 1.0 lb/cubic inch is used to model incompressible, viscous flows in all three cases. Increasing values of volumetric expansion coefficients are used for the problems: 1.0E+03 and 1.0E+04 in/°F, which also represent the Rayleigh numbers for problems e9x7a and e9x7b, respectively. Fluid thermal conductivity of 1.0 Btu/sec/in/°F. is used in all cases.

Results

The vector plots of fluid velocity fields for problems e9x7a and e9x7b are given in Figures 9.7-1 and 9.7-2, respectively. The circulating flow fields, as shown by the arrows that are scaled according to their magnitudes, tend to reach an oval pattern as the Rayleigh number gets higher. The resulting temperature distributions in the fluids are given by the contour plots in Figures 9.7-3 and 9.7-4 for problems e9x7a and e9x7b, respectively. Higher Rayleigh numbers produce more significant effects of natural convection.

Parameters, Options, and Subroutines Summary

Examples e9x7a.dat and e9x7b.dat:

Parameters	Model Definition Options
DIST LOADS	CONNECTIVITY
ELEMENTS	COORDINATES
END	CONTINUE
FLUID	CONTROL
SIZING	DIST LOADS
	END OPTION
	FIXED VELOCITY
	ISOTROPIC
	PARAMETERS
	POST
	PRINT ELEMENT
	STEADY STATE

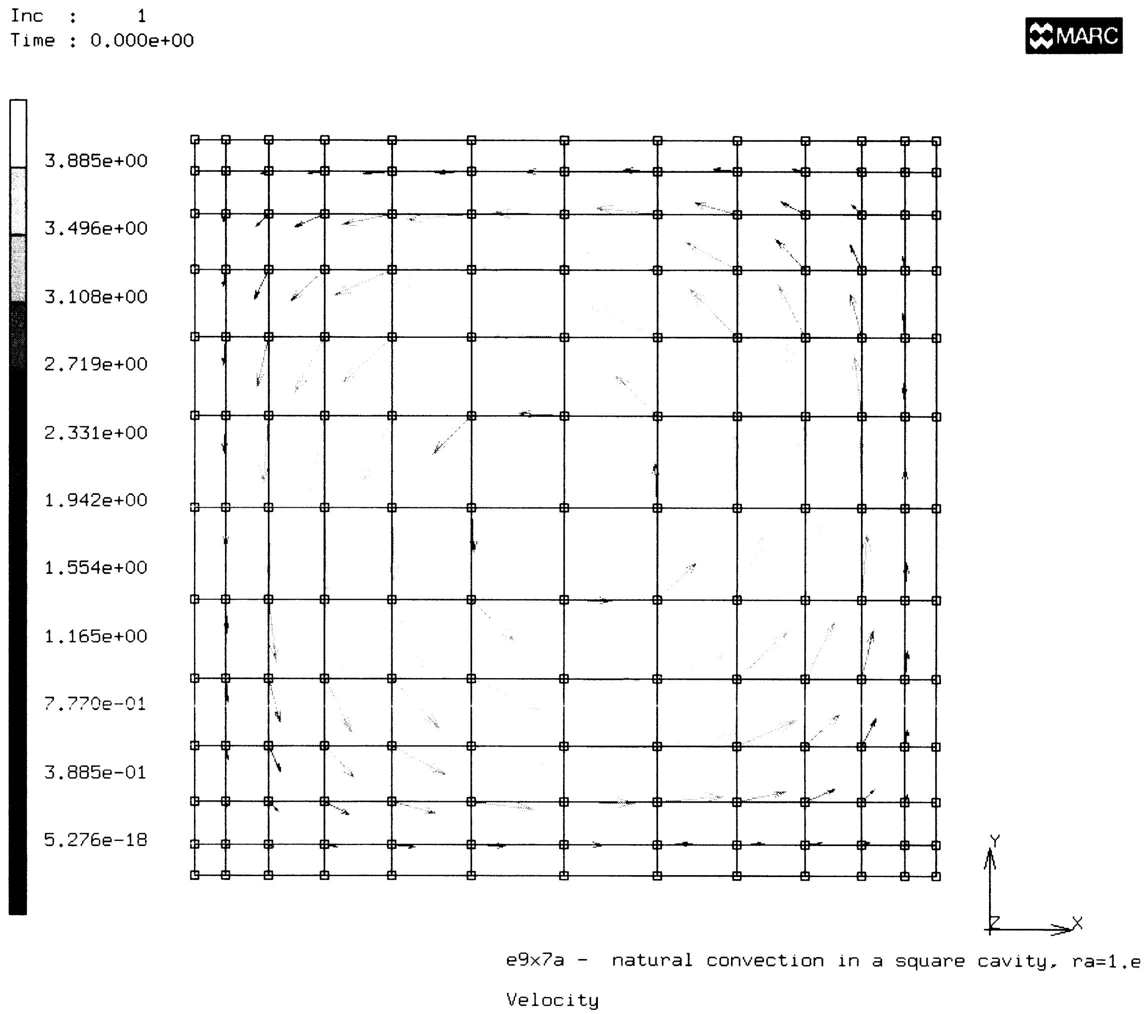


Figure 9.7-1 Vector Plot of the Natural Convective Flow Velocity Field with Rayleigh Number = 1.0e+03, Discretized using Element Type 11 and the Penalty Method

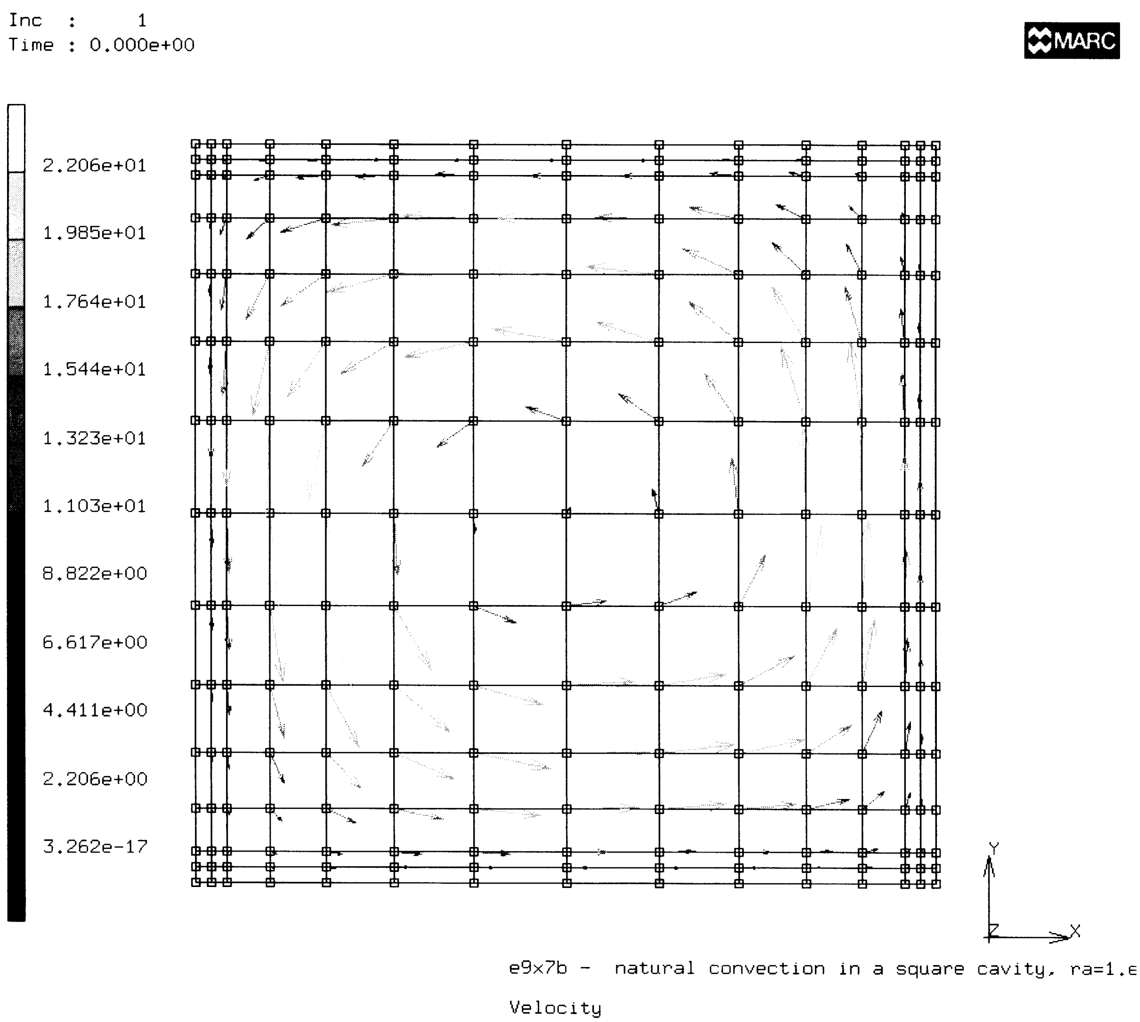


Figure 9.7-2 Vector Plot of the Natural Convective Flow Velocity Field with Rayleigh Number = 1.0e+04, Discretized using Element Type 11 and the Penalty Method

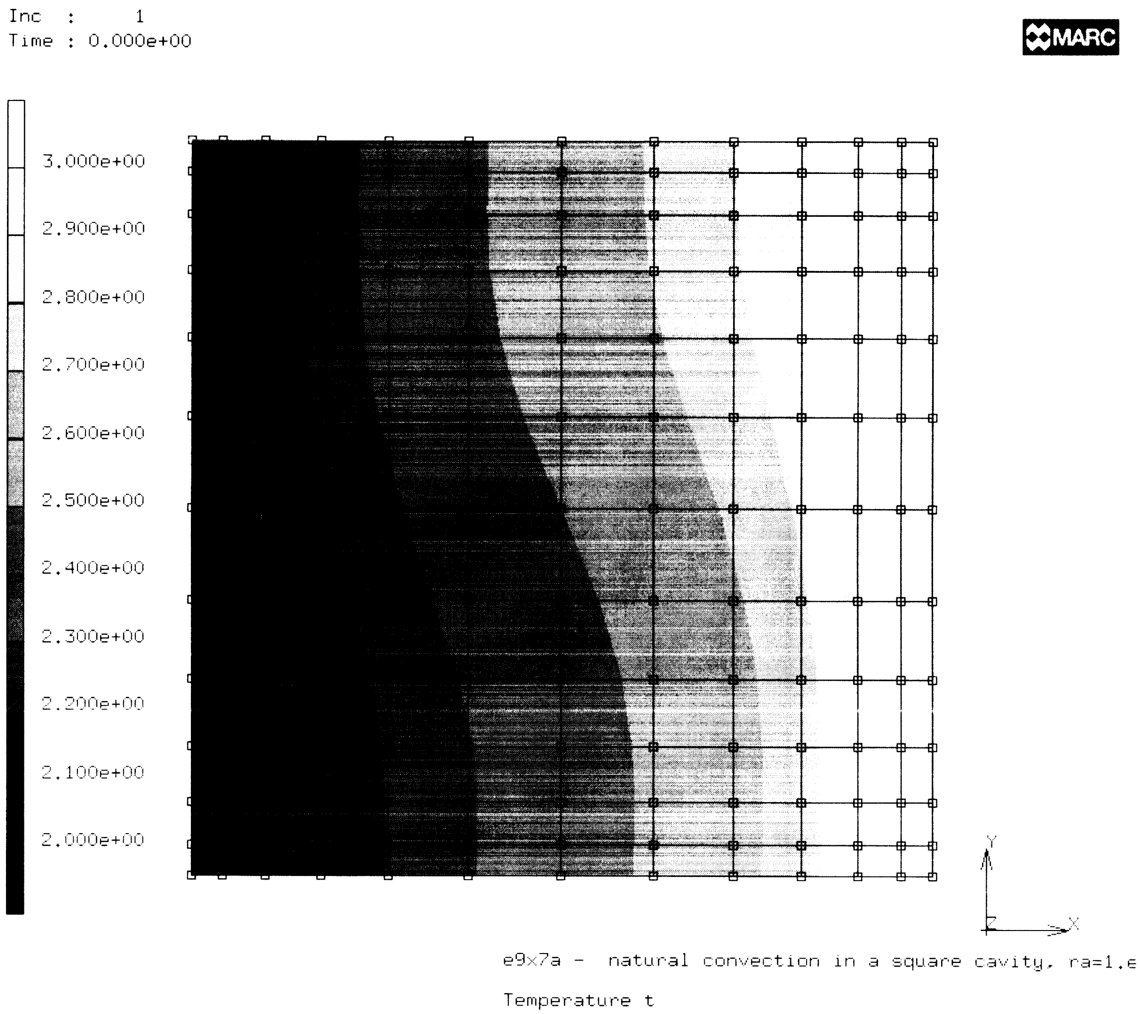


Figure 9.7-3 Contour Plot of the Natural Convective Flow Temperature Field with Rayleigh Number = 1.0e+03, Discretized using Element Type 11 and the Penalty Method

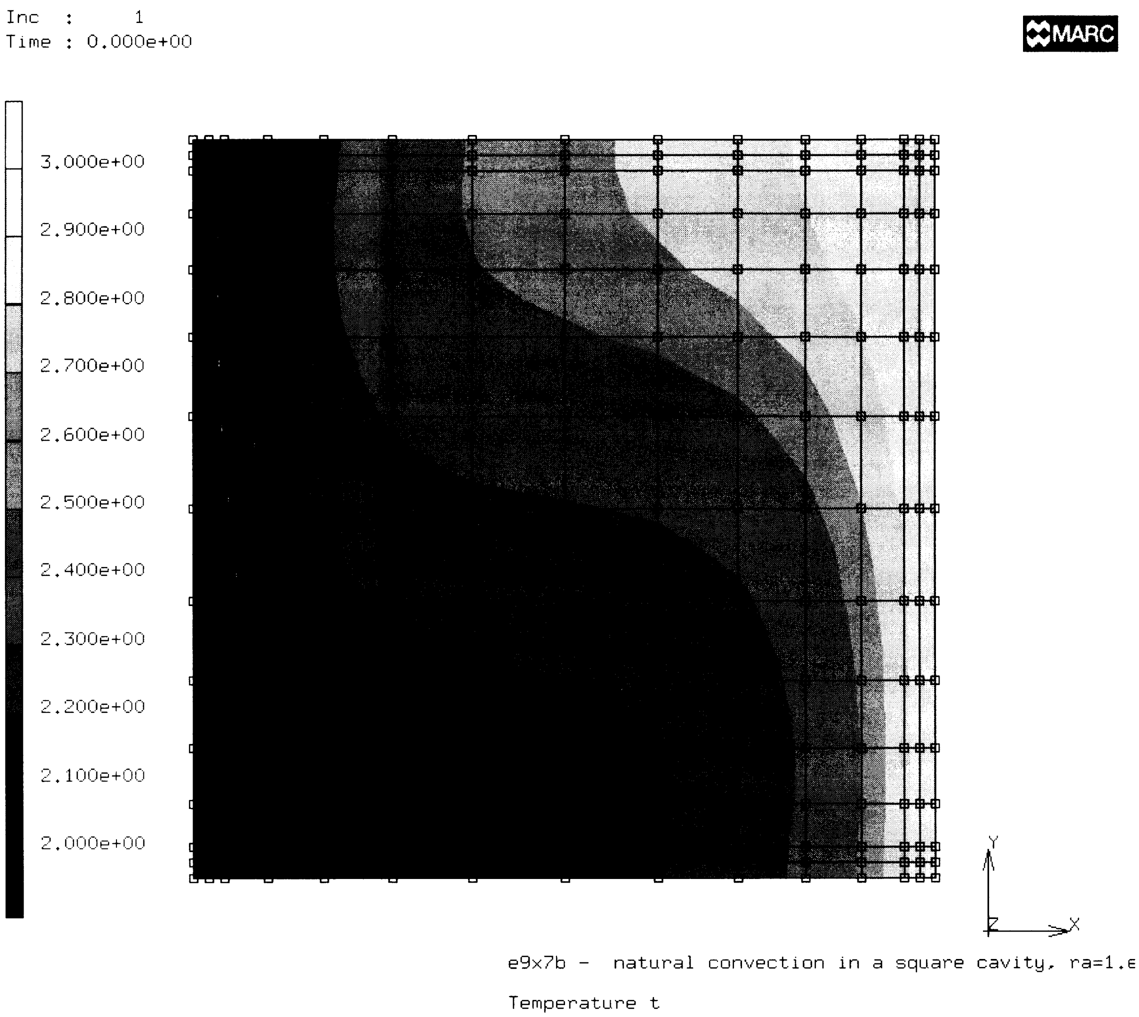


Figure 9.7-4 Contour Plot of the Natural Convective Flow Temperature Field with Rayleigh Number = 1.0e+04, Discretized using Element Type 11 and the Penalty Method

9.8 Flow Around Tubes

This problem demonstrates a modeling of viscous, incompressible flow around obstacles represented by a group of tubes. For this steady state approximation of isothermal flow using Navier-Stokes equations, the results obtained using the mixed method is presented.

Element

Element type 11 is a lower-order, 4-node, bilinearly interpolated, planar element used in this problem to discretize the fluid domain. Using the mixed method, each node has three degrees of freedom: two planar velocity components and a pressure.

Model

The rectangular eight-by-one square inches fluid domain is intersected by three rigid tubes crossing in the direction perpendicular to the domain. Each tube has a diameter of 1.0 inch. Only upper- or lower-half cross section of the tubes are cutting out the fluid domain. The finite element mesh is given in Figure 9.8-1, which consists of a total of 980 elements.

Boundary Conditions

All sections along the perimeter of the fluid domain, except for the openings for fluid inflow and outflow on the extreme left and right of the model, respectively, are considered rigid walls. No-slip boundary conditions along the walls requires that both components of fluid velocity along the sections be specified as zero. At the inflow section on the left, velocity of 1.0 inch per second is applied along the positive x-direction. The prescribed velocity pushes the fluid to flow through the obstacles created by the tubes. Velocity component at the y-direction is set to zero on both the inflow and the outflow sections along the perimeter.

Material

Newtonian fluid material with viscosity of 0.01 lbf.sec./square inch and a mass density of 1.0 lb/cubic inch is used to model the viscous flow. Reynolds number of this problem is 57.72.

Results

The contour plots of velocity (magnitude) and pressure distributions are given by Figures 9.8-2 and 9.8-3, respectively.

Parameters, Options, and Subroutines Summary

Examples e9x8.dat:

Parameters	Model Definition Options
DIST LOADS	CONNECTIVITY
ELEMENTS	COORDINATES
END	CONTINUE
FLUID	CONTROL
SIZING	END OPTION
	FIXED VELOCITY
	ISOTROPIC
	POST
	STEADY STATE

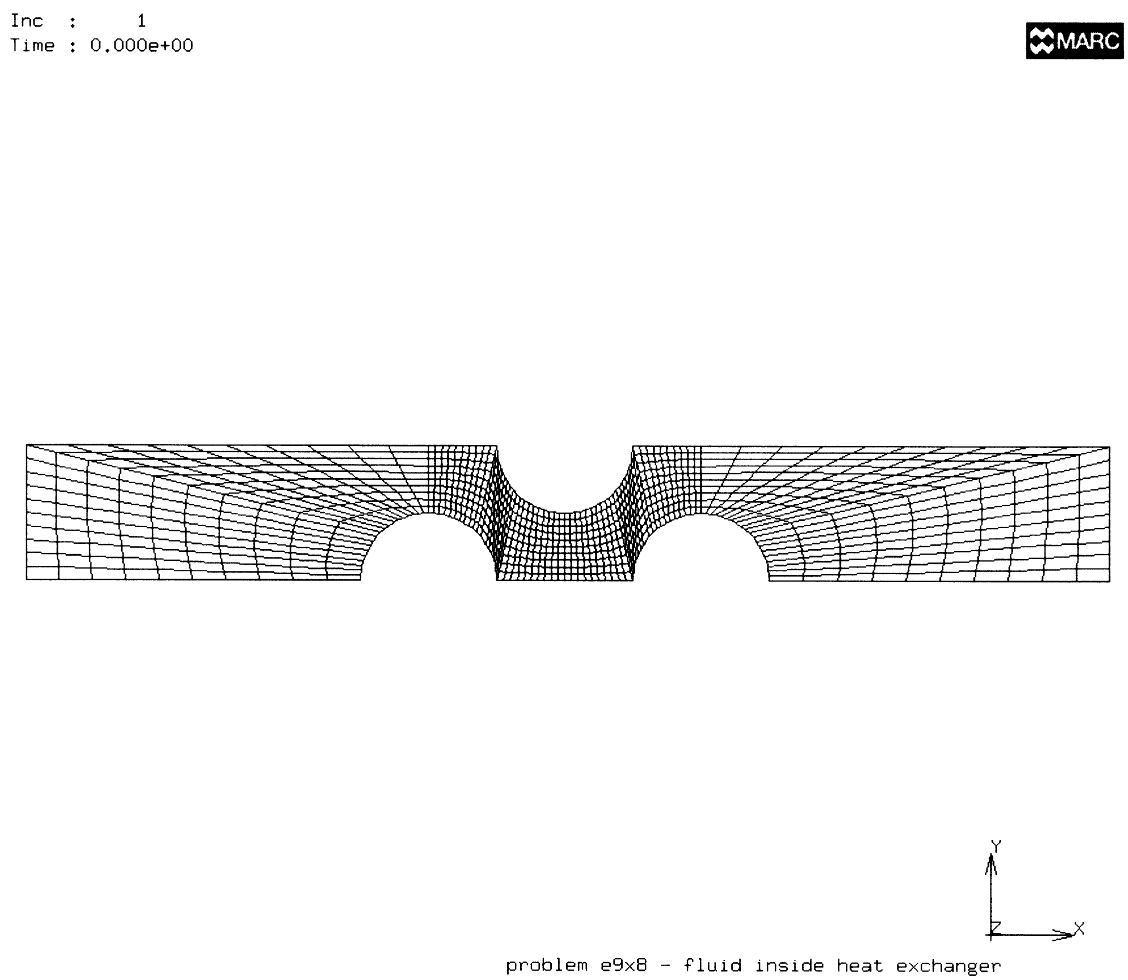


Figure 9.8-1 Finite Element Mesh for the Flow Over Cylinders

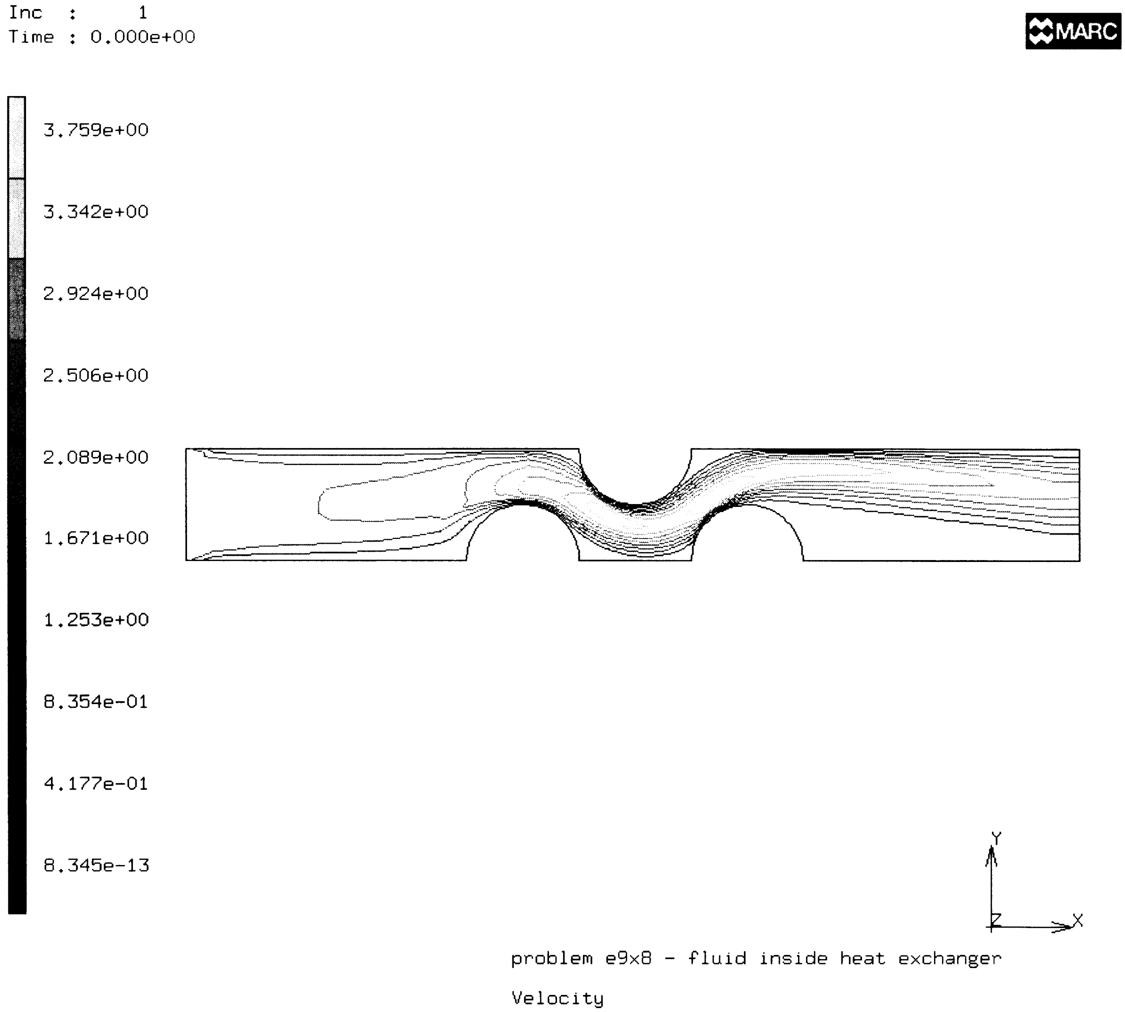


Figure 9.8-2 Contour Plot of the Flow Over Cylinders Velocity Field, Discretized using Element Type 11 and the Mixed Method

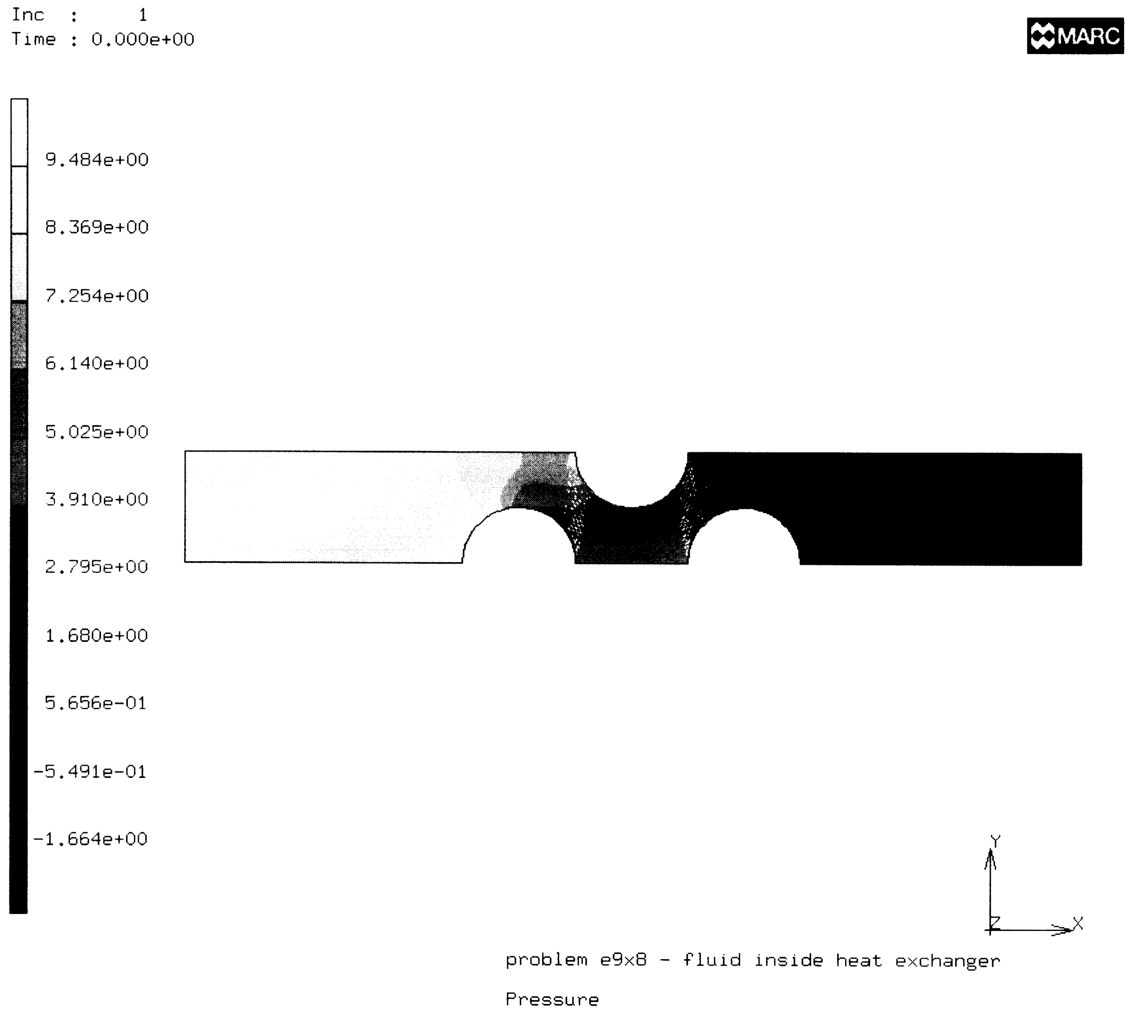
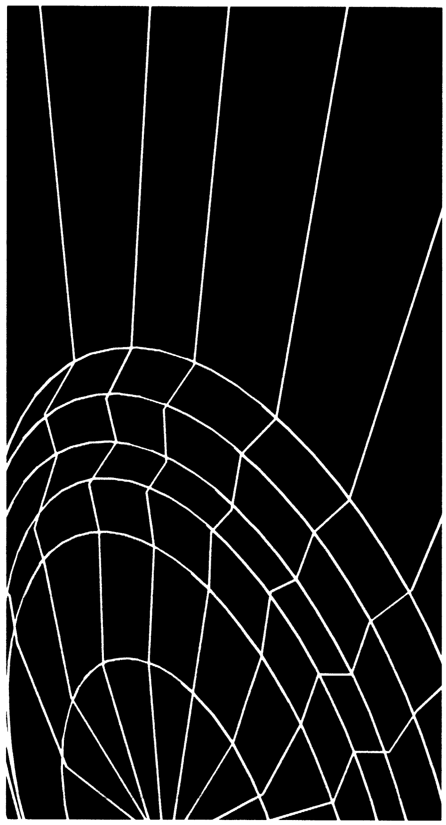
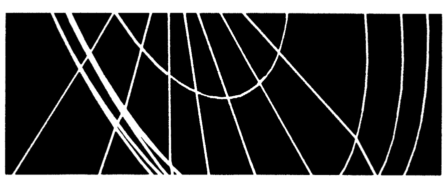


Figure 9.8-3 Contour Plot of the Flow Over Cylinders Pressure Field, Discretized using Element Type 11 and the Mixed Method

MARC
Analysis
Research
Corporation



MARC



Volume E

Demonstration Problems

Version K7

Chapter 10
Design Sensitivity and Optimization

10 Design Sensitivity and Optimization Contents

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10 Design Sensitivity and Optimization List of Figures

Figure **Page**

10 Design Sensitivity and Optimization

Design sensitivity analysis and design optimization features have been available in MARC starting with Version K7.1. Substantial information about these features is now available in *MARC Volume A: Theory and User Information* and *MARC Volume C: Program Input*, as well as in the *MENTAT 3.1-MARC K7.1 New Features User Guide*, with supplemental material as it relates to various elements in *MARC Volume B: Element Library.*

Briefly summarized, the sensitivity analysis feature is useful in obtaining gradients of prescribed response quantities with respect to user-defined design variables, selectable from an available set. It is also useful in obtaining the finite element contributions to these prescribed response quantities. The sensitivity analysis feature is applicable to the fixed design submitted in a MARC data file.

The design optimization feature is useful in attempting to minimize a design objective, such as material mass, by variation of selected design variables, while adhering to limitations imposed on the response as well as on the values of the design variables. Since the design is going to change, the prescribed values of the design variables that appear in the MARC data file are replaced by those due to a varying design. Thus, for example, if a particular plate thickness is a design variable, then the value of that thickness given in the GEOMETRY option is no longer of interest (although a future algorithm may use it as a starting design). Instead, a series of values are generated for this thickness, and for any other design variables, based on their lower and upper bounds and on the flow of the design optimization algorithm.

The sensitivity analysis feature allows the plotting of the gradients and the element contributions, whereas the design optimization feature allows the generation of history plots for the changes in the objective function and in the design variables. It also allows the plotting of the design variable values at the end of any cycle of optimization. An important item to remember is that for sensitivity analysis, the sensitivity results are given as subincrements of the last increment, whereas for design optimization the optimization cycles are given as subincrements of the zeroth increment. For analysis purposes, the zeroth increment should be a dummy increment when design sensitivity analysis or design optimization is to be performed.

During design sensitivity analysis, any eigenfrequency analysis or applied load cases are given as increments 1 through, say, N. The N+1'th increment is that for which the M subincrements are sensitivity analysis results for M prescribed nontrivial response quantities. The results of K cycles of design optimization are in the K subincrements of increment 0, the later increments being the results for the analysis of the best design obtained, for any prescribed eigenfrequency analysis and all applied load cases.

The results of both types of analyses are well documented numerically in the output files and you should refer to these as well as to the log file, in addition to the graphic representations obtainable via the interpretation of the post files through Mentat. An example of an important reason for this is that the graphic representations may show a small derivative to be insignificant in comparison to other derivatives, whereas that derivative may actually be quite important due to differences in the orders of magnitudes of the design variables.

The seven problems in this chapter treat different types of elements and response as well as different design variables and constraints. Thus, hopefully, they are a representative set in terms of the features offered to the user. Sensitivity analysis and design optimization are performed by means of different data files, which usually, but not always, differ only by the parameter data blocks "design sensitivity" and "design optimization". The design sensitivity data files are e10x1a.dat through e10x7a.dat, whereas the design optimization data files are e10x1b.dat through e10x7b.dat.

Table 10-1 summarizes the element type and options used in these demonstration problems.

Table 10-1 Design Sensitivity and Design Optimization Demonstration Problems

Problem Number	Element Type(s)	Parameters	Model Definition	History Definition	User Subroutines	Problem Description
10.1 (a)	52	DESIGN SENSITIVITY DYNAMIC	DESIGN DISPLACEMENT CONSTRAINTS DESIGN FREQUENCY CONSTRAINTS DESIGN OBJECTIVE DESIGN STRESS CONSTRAINTS DESIGN VARIABLES MASSES TYING	MODAL SHAPE POINT LOAD	—	Beam-column frame sensitivity analysis.
10.1 (b)	52	DESIGN OPTIMIZATION DYNAMIC	DESIGN DISPLACEMENT CONSTRAINTS DESIGN FREQUENCY CONSTRAINTS DESIGN OBJECTIVE DESIGN STRESS CONSTRAINTS DESIGN VARIABLES MASSES TYING	MODAL SHAPE POINT LOAD	—	Beam-column frame design optimization.
10.2 (a)	26	DESIGN SENSITIVITY	DESIGN OBJECTIVE DESIGN STRAIN CONSTRAINTS DESIGN VARIABLES	DIST LOADS POINT LOAD	—	Plane stress sensitivity analysis.
10.2 (b)	26	DESIGN OPTIMIZATION	DESIGN OBJECTIVE DESIGN STRAIN CONSTRAINTS DESIGN VARIABLES	DIST LOADS POINT LOAD	—	Plane stress design optimization.
10.3 (a)	21	DESIGN SENSITIVITY DYNAMIC	DESIGN DISPLACEMENT CONSTRAINTS DESIGN FREQUENCY CONSTRAINTS DESIGN OBJECTIVE DESIGN STRESS CONSTRAINTS DESIGN VARIABLES	DIST LOADS MODAL SHAPE	—	Thick plate (brick elements) design sensitivity.

Table 10-1 Design Sensitivity and Design Optimization Demonstration Problems (Continued)

Problem Number	Element Type(s)	Parameters	Model Definition	History Definition	User Subroutines	Problem Description
10.3 (b)	21	DESIGN OPTIMIZATION DYNAMIC	DESIGN DISPLACEMENT CONSTRAINTS DESIGN FREQUENCY CONSTRAINTS DESIGN OBJECTIVE DESIGN STRESS CONSTRAINTS DESIGN VARIABLES	DIST LOADS MODAL SHAPE	—	Thick plate (brick elements) design optimization.
10.4 (a)	75	DESIGN SENSITIVITY DYNAMIC SHELL SECT	DESIGN FREQUENCY CONSTRAINTS DESIGN OBJECTIVE DESIGN STRESS CONSTRAINTS DESIGN VARIABLES	MODAL SHAPE POINT LOAD	—	Shell roof design sensitivity.
10.4 (b)	75	DESIGN OPTIMIZATION DYNAMIC SHELL SECT	DESIGN FREQUENCY CONSTRAINTS DESIGN OBJECTIVE DESIGN STRESS CONSTRAINTS DESIGN VARIABLES	MODAL SHAPE POINT LOAD	—	Shell roof design optimization.
10.5 (a)	75	DESIGN SENSITIVITY DYNAMIC	COMPOSITE DESIGN DISPLACEMENTS CONSTRAINTS DESIGN OBJECTIVE DESIGN STRAIN CONSTRAINTS DESIGN STRESS CONSTRAINTS DESIGN VARIABLES ORIENTATION ORTHOTROPIC	POINT LOAD	—	Composite plate design sensitivity.

Table 10-1 Design Sensitivity and Design Optimization Demonstration Problems (Continued)

Problem Number	Element Type(s)	Parameters	Model Definition	History Definition	User Subroutines	Problem Description
10.5 (b)	75	DESIGN OPTIMIZATION DYNAMIC	COMPOSITE DESIGN DISPLACEMENTS CONSTRAINTS DESIGN OBJECTIVE DESIGN STRAIN CONSTRAINTS DESIGN STRESS CONSTRAINTS DESIGN VARIABLES ORIENTATION ORTHOTROPIC	POINT LOAD	—	Composite plate design optimization.
10.6 (a)	9	DESIGN SENSITIVITY	DESIGN OBJECTIVE DESIGN STRESS CONSTRAINTS DESIGN VARIABLES	POINT LOAD	—	Planar truss design sensitivity.
10.6 (b)	9	DESIGN OPTIMIZATION	DESIGN OBJECTIVE DESIGN STRESS CONSTRAINTS DESIGN VARIABLES	POINT LOAD	—	Planar truss design optimization.
10.7 (a)	14	DESIGN SENSITIVITY DYNAMIC	DESIGN DISPLACEMENTS CONSTRAINTS DESIGN FREQUENCY CONSTRAINTS DESIGN OBJECTIVE DESIGN STRESS CONSTRAINTS DESIGN VARIABLES MASSES TYING	MODAL SHAPE POINT LOAD	—	Beam-column frame sensitivity analysis.
10.7 (b)	14	DESIGN OPTIMIZATION DYNAMIC	DESIGN DISPLACEMENTS CONSTRAINTS DESIGN FREQUENCY CONSTRAINTS DESIGN OBJECTIVE DESIGN STRESS CONSTRAINTS DESIGN VARIABLES MASSES TYING	MODAL SHAPE POINT LOAD	—	Beam-column frame design optimization.

10.1 Design Sensitivity Analysis and Optimization of an Alternator Mount Frame using Element 52

A spatial frame representing the support of an alternator is considered. Design sensitivity and optimization of the system are performed with constraints on static response under two separate load cases and on eigenfrequencies.

Element

Element 52, a straight Euler-Bernoulli beam in space with linear elastic response, is used. The element has six coordinates per node: the first three are (x,y,z) global coordinates of the system, the other three are the global coordinates of a point in space which locates the local x-axis of the cross section.

Model

The 3D frame is modeled using 16 beam-column elements and 20 nodes. The columns are clamped at the base. The elements have arbitrary solid cross sections. Two masses are lumped in the middle of two horizontal beams at nodes 14 and 18 (Figure 10.1-1). Elements numbered 1 through 8 are the columns and the rest of the elements are the beams. The beam to column connections are obtained through tying of separately numbered nodes.

Geometry

The column elements are 250 cm long; the beam elements in the x-direction are 192.5 cm long and those in the z-direction are 157.5 cm. The column cross-sectional areas are 8625 cm whereas the beams have 6625 cm cross sections. I_{xx} and I_{yy} are the same for all elements: 9.5 x 10^6 and 4.0 x 10^6, respectively.

Element 52 computes the torsional stiffness of the section as:

$$K_t = \frac{E}{2(1+\nu)}(I_{xx} + I_{yy})$$

Then, in order to obtain the correct stiffness, an artificial Poisson's ratio ν^* is chosen to be used only for this purpose. See **Material Properties** below.

Material Properties

The material is assumed to be linearly elastic, homogeneous, and isotropic. Young's modulus is $E = 2.5 \times 10^8$ kg/cm sec^2 and the mass density is $\rho = 2.55\ 10^{-3}$ kg/cm^3. The Poisson's ratio is an artificial one since it is used here to compute torsional stiffness only. The value of such an artificial Poisson's ratio normally depends on the actual type of cross section used (see problem 6.10). However, currently MARC does not modify this ratio with changes in the cross section. The lumped masses are M = 19000 kg each.

Design Variables and Objective Function

For this problem, there are three design variables of the geometry type: the cross-sectional area A of the beam, the moment of inertia I_{xx} of the columns, and the moments of inertia I_{yy} of the beams in that order as variables 1, 2, and 3. The objective function for this problem is the total mass of the material used, which means that the design optimization procedure seeks to minimize the mass. The variables are linked over all beams or all columns, as applicable.

Design Constraints

The design constraints in this analysis includes stress constraints, displacement constraints, and frequency constraints. Stress constraints are imposed on generalized stresses in all of the elements. Displacement constraints consist of a limit on the translation along the first degree of freedom at node 15 under the first static load case, and a limit on the rotation about the first degree of freedom at node 19 under the second static load case. Frequency constraints are on the fundamental frequency and on the difference between the frequencies of the first two modes.

Results

The design sensitivity and design optimization cases are run as separate jobs, with the data files e10x1a.dat and e10x1b.dat, respectively. Figure 10.1-2 shows the gradient of the maximum second generalized stress (bending moment about x-axis) for element 2 under load case 2 (first static load case) with respect to all design variables. Figure 10.1-3 is a plot showing, on the finite element model, the element contributions to the response quantity in question.The gray scale being useless for frames, the unaveraged values are shown on the elements using an alphabetical scale. Figure 10.1-4 shows the change in the objective function with the optimization cycle in the form of a history plot. Figure 10.1-5 is a bar chart showing the values of the design variables at the best feasible (F) design obtained. Since the cross-sectional area value is several orders of magnitude smaller than the moments of inertia, a fitted plot shows the first variable value as the initial y-coordinate.

Parameters, Options. and Subroutines Summary

Listed below are the options used in example e10x1a.dat:

Parameters	Model Definition Options	History Definition Options
DESIGN SENSITIVITY	CONNECTIVITY	CONTINUE
DYNAMIC	COORDINATES	MODAL SHAPE
ELEMENTS	DESIGN DISPLACEMENT CONSTRAINTS	POINT LOAD
END	DESIGN FREQUENCY CONSTRAINTS	
SIZING	DESIGN OBJECTIVE	
TITLE	DESIGN STRESS CONSTRAINTS	
	DESIGN VARIABLES	
	END OPTION	
	FIXED DISP	
	GEOMETRY	
	ISOTROPIC	
	MASSES	
	POINT LOAD (dummy)	
	POST	
	TYING	

Listed below are the options used in example e10x1b.dat:

Parameters	Model Definition Options	History Definition Options
DESIGN OPTIMIZATION	CONNECTIVITY	CONTINUE
DYNAMIC	COORDINATES	MODAL SHAPE
ELEMENTS	DESIGN DISPLACEMENT CONSTRAINTS	POINT LOAD
END	DESIGN FREQUENCY CONSTRAINTS	
SIZING	DESIGN OBJECTIVE	
TITLE	DESIGN STRESS CONSTRAINTS	
	DESIGN VARIABLES	
	END OPTION	
	FIXED DISP	
	GEOMETRY	
	ISOTROPIC	
	MASSES	
	POINT LOAD (dummy)	
	POST	
	TYING	

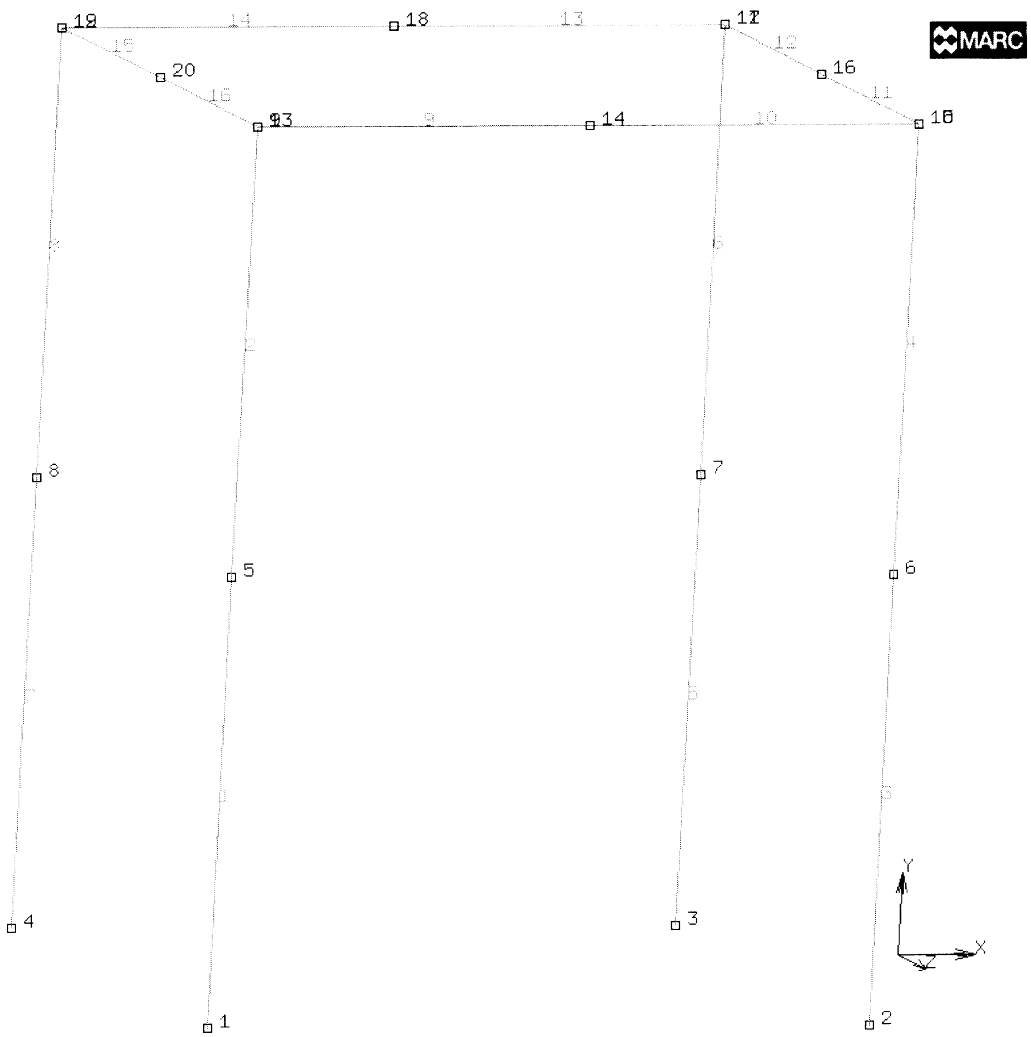

Figure 10.1-1 Alternator Mount Frame Model using Element 52

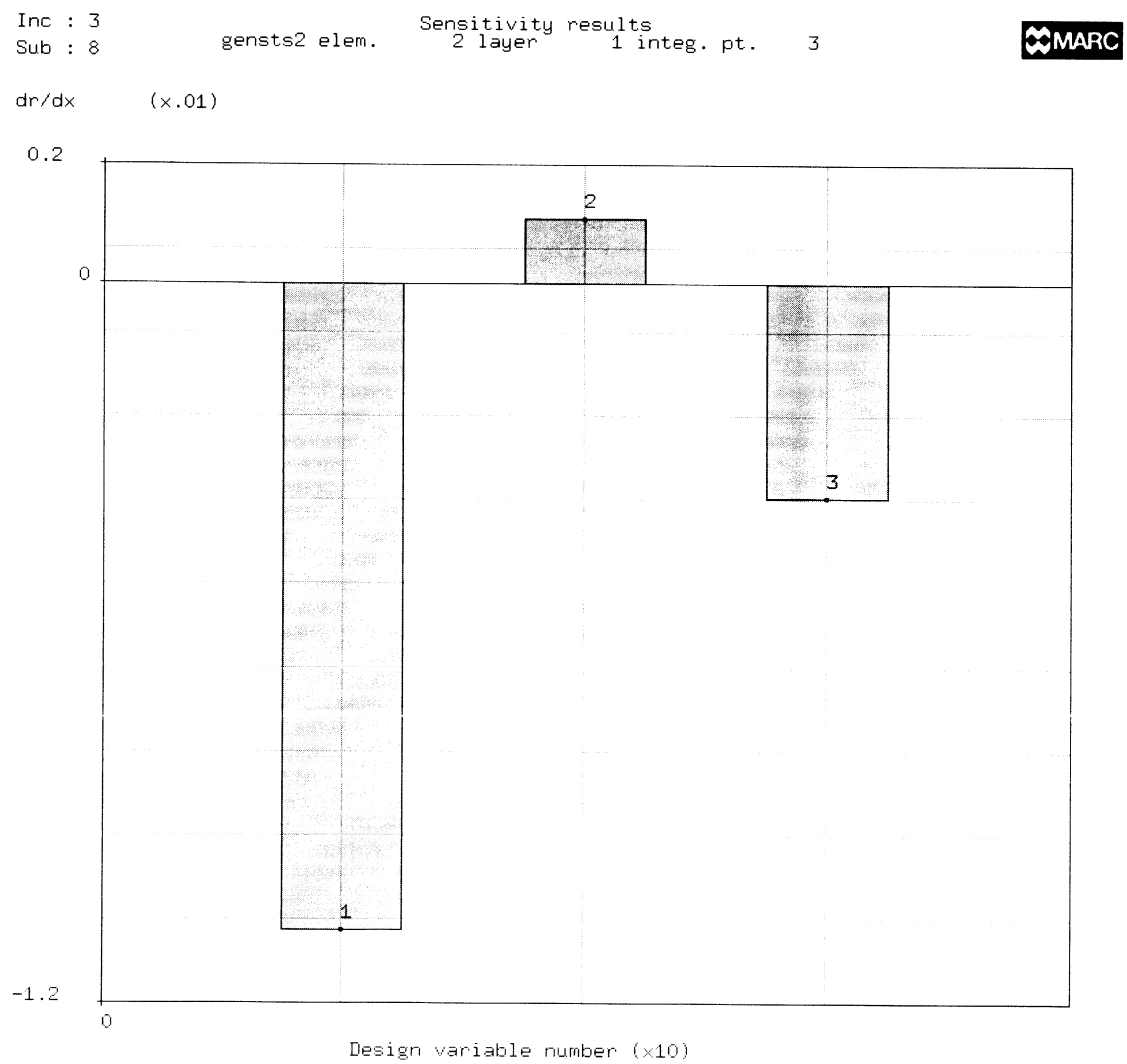

Figure 10.1-2 Gradient of Maximum M_x (Element 2) with Respect to Design Variables, Load Case 2

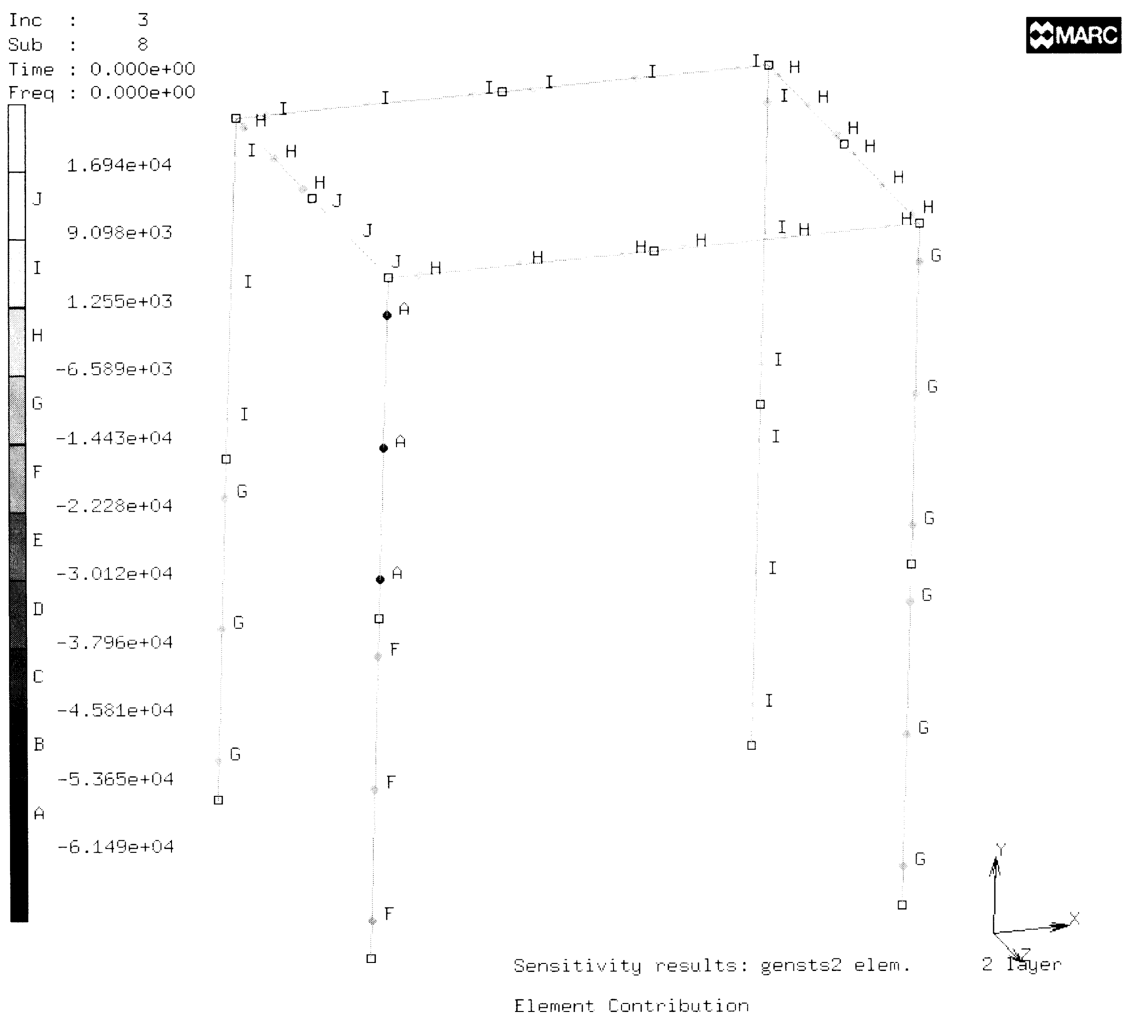

Figure 10.1-3 Element Contributions to Response of Figure 10.1-2

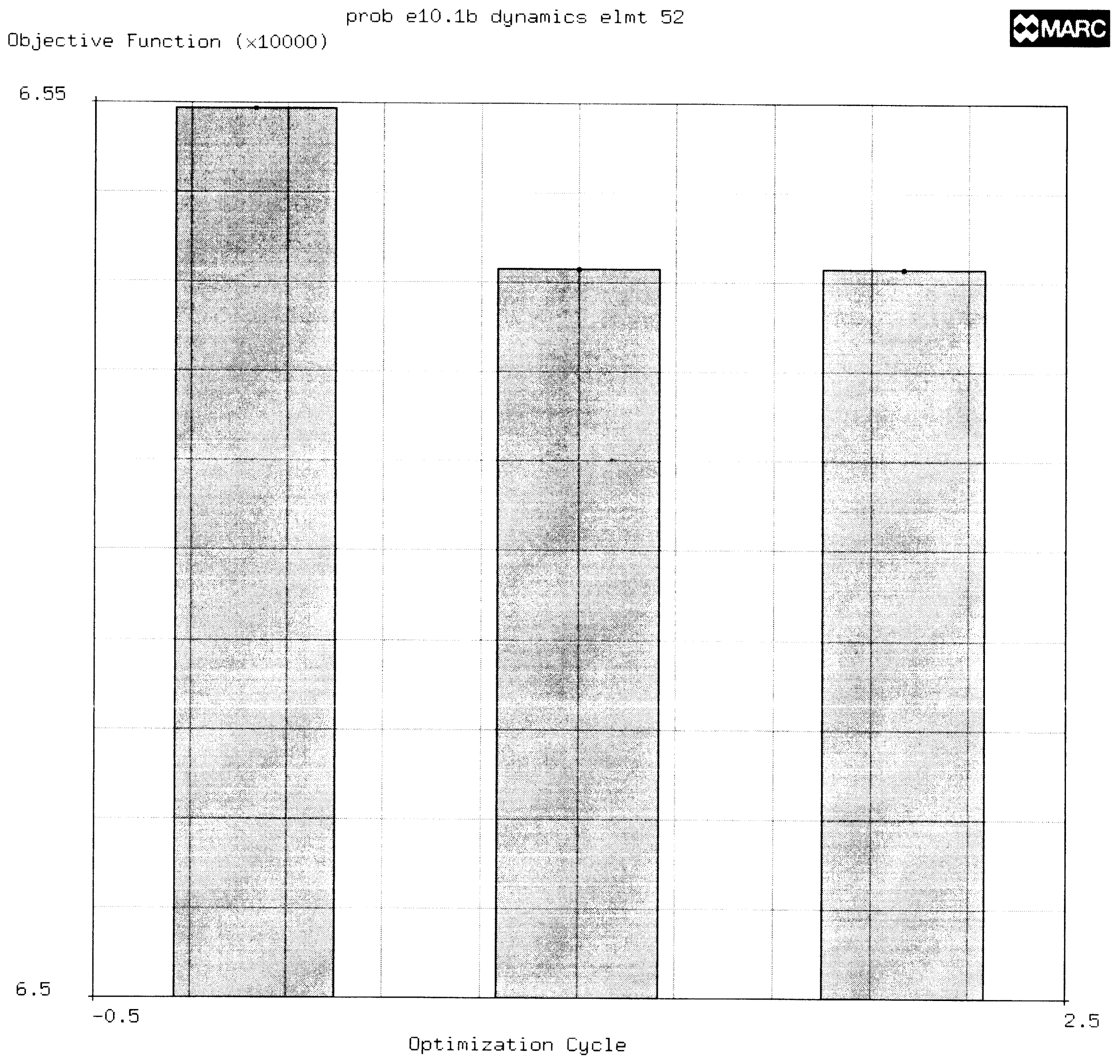

Figure 10.1-4 History Plot of the Objective Function

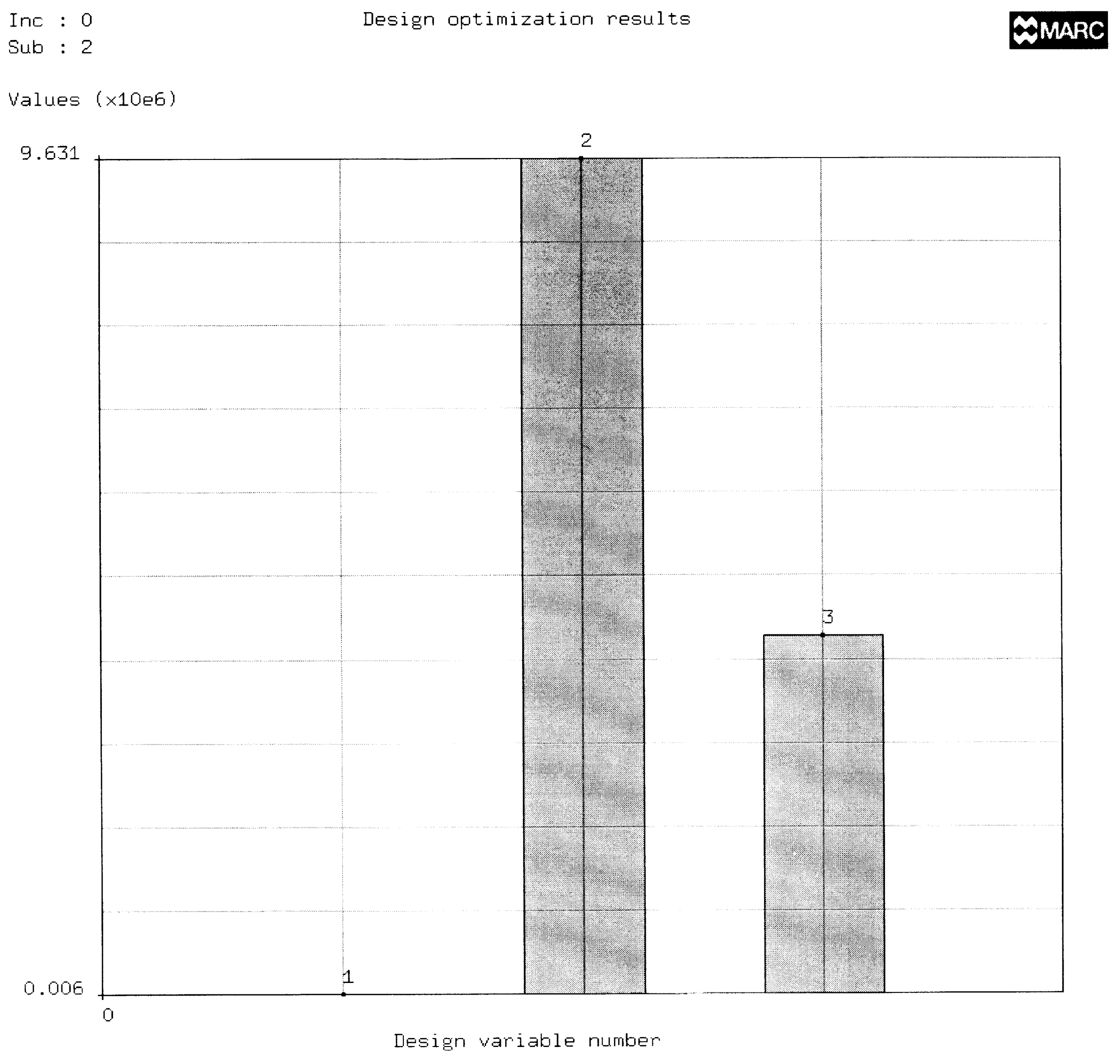

Figure 10.1-5 Design Variables at Best Feasible Design (Cycle 2)

10.2 Design Sensitivity Analysis and Optimization of a Plate with a Hole

The design sensitivity and design optimization features are applied to the problem of a square plate with a circular hole (Timoshenko and Goodier, *Theory of Elasticity*). The second order isoparametric plane stress element (type 26) is used.

Elements

Element type 26 is a second-order isoparametric plane stress element. It is an 8-noded quadrilateral.

Model

The dimensions of the plate are 10 inches square with a 2 inch radius central hole. Only one quarter of the plate is modeled due to symmetry conditions. The finite element mesh for this model is shown in Figure 10.2-1. The elements near the hole are smaller to capture the strain variation. There are 20 quadrilateral elements in the mesh.

Material Properties

The material for all of the elements is elastic and isotropic with a Young's modulus of 30.0E+06 psi and a Poisson's ratio (ν) of 0.3.

Geometry

The thicknesses prescribed for the purpose of sensitivity analysis are: 0.7 inch for elements 1 through 4 and 11 through 14; 1.0 inch for the rest.

Loads and Boundary Conditions

The structure is acted upon by two separate load cases. The first load case is a distributed load applied to the top edge of the quarter model. Point loads, acting horizontally along the nodes on the right edge of the mesh, represent the second load case. The boundary conditions are determined by the symmetry conditions and require that the nodes along $y = 0$ axis have no vertical displacement, and the nodes along the $x = 0$ axis have no horizontal displacement. The origin of the model is at the center of the hole.

Design Variables and Objective Function

There are three types of design variables employed: plate thickness (variables 1 and 2), Poisson's ratio (variable 3), and Young's modulus (variable 4). For the plate thickness, elements are linked in the same two groups as for the prescribed thicknesses. The first design variable links the 8 larger elements which are farther away from the hole, and the second design variable links the remaining

12 smaller elements. The objective function is the volume of the material. Therefore, the gradient of this function is to be obtained during sensitivity analysis. For design optimization, the total material volume is to be minimized.

Design Constraints

For this problem, only strain constraints are applied. These include the two independent normal strains, the in-plane shear strain, the von Mises strain, and the maximum absolute valued principal strain. These constrains are applied to the same element (number 11) for both load cases. For the first normal strain and the shear strain, the constraints are on the absolute value.

Results

The design sensitivity and design optimization cases are run as separate jobs, with the data files e10x2a.dat and e10x2b.dat, respectively. Figure 10.2-2 shows the gradient of the maximum von Mises strain over the integration points of element 11 under load case 2 with respect to all design variables. While the derivative with respect to the Young's modulus E is very small (-0.9×10^{-12}), and appears as zero in Figure 10.2-2, it should be kept in mind that E is many orders of magnitude greater than the other variables. Thus, the small derivative may not necessarily be considered trivial. Figure 10.2-3 is a contour plot showing, on the finite element model, the element contributions to the response quantity in question. Figure 10.2-4 shows the change in the objective function with the optimization cycle in the form of a history plot. It is noted that the best feasible (F) design is obtained at cycle 9. Figure 10.2-5 is a bar chart showing the values of the first three design variables at the best feasible design obtained. The value of E does not change from the starting vertex value of 3.1425×10^8, and, therefore, it is not plotted here so that the other three values can be seen.

Parameters, Options. and Subroutines Summary

Listed below are the options used in example e10x2a.dat:

Parameters	Model Definition Options	History Definition Options
DESIGN SENSITIVITY	CONNECTIVITY	CONTINUE
ELEMENTS	COORDINATES	DIST LOADS
END	DESIGN OBJECTIVE	POINT LOAD
SIZING	DESIGN STRAIN CONSTRAINTS	
TITLE	DESIGN VARIABLES	
	DIST LOADS (dummy)	
	END OPTION	
	FIXED DISP	
	GEOMETRY	
	ISOTROPIC	
	OPTIMIZE	
	POINT LOAD (dummy)	
	POST	

Listed below are the options used in example e10x2b.dat:

Parameter Options	Model Definition Options	History Definition Options
DESIGN OPTIMIZATION	CONNECTIVITY	CONTINUE
ELEMENTS	COORDINATES	DIST LOADS
END	DESIGN OBJECTIVE	POINT LOAD
SIZING	DESIGN STRAIN CONSTRAINTS	
TITLE	DESIGN VARIABLES	
	DIST LOADS (dummy)	
	END OPTION	
	FIXED DISP	
	GEOMETRY	
	ISOTROPIC	
	OPTIMIZE	
	POINT LOAD (dummy)	
	POST	

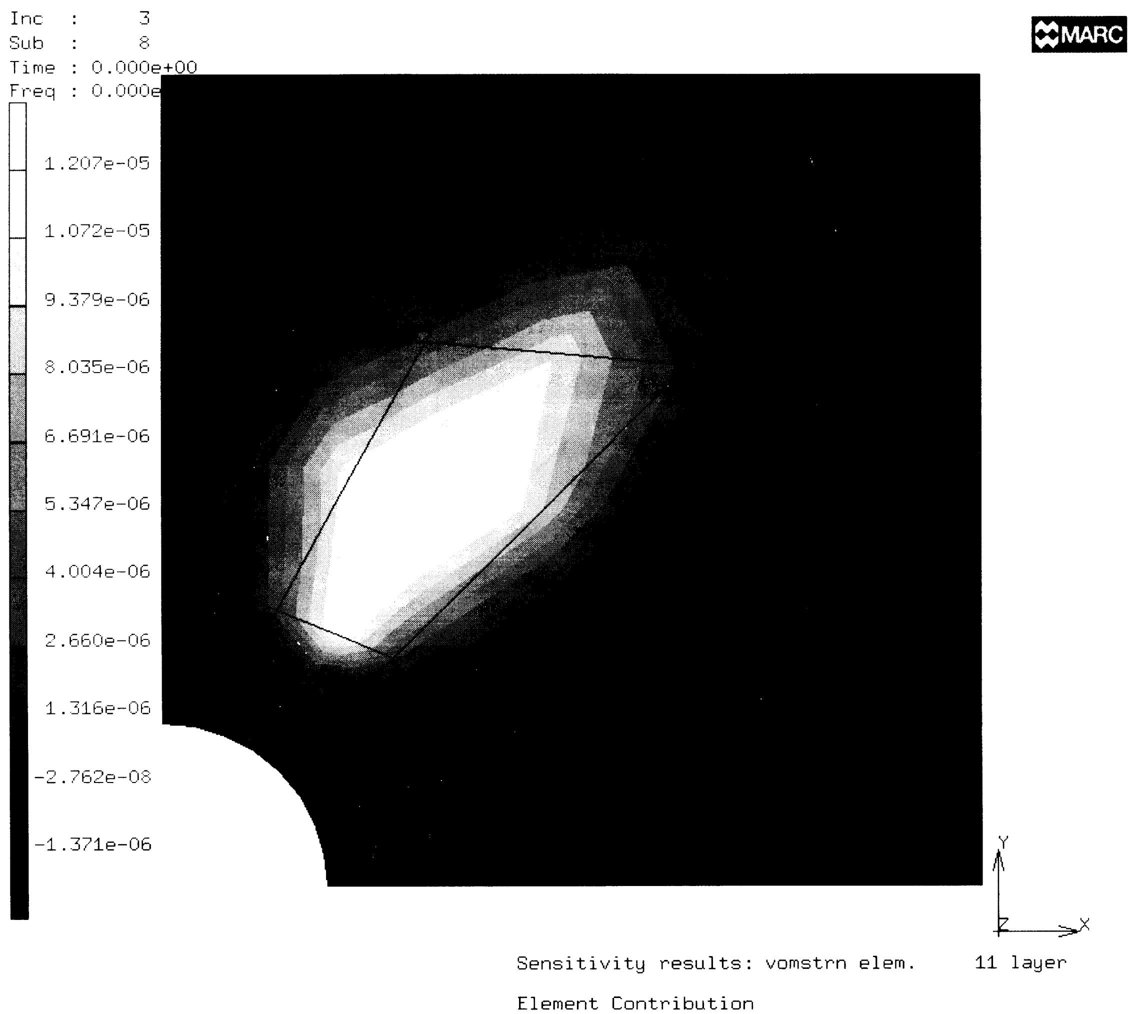

Figure 10.2-3 Element Contributions to Response of Figure 10.2-2

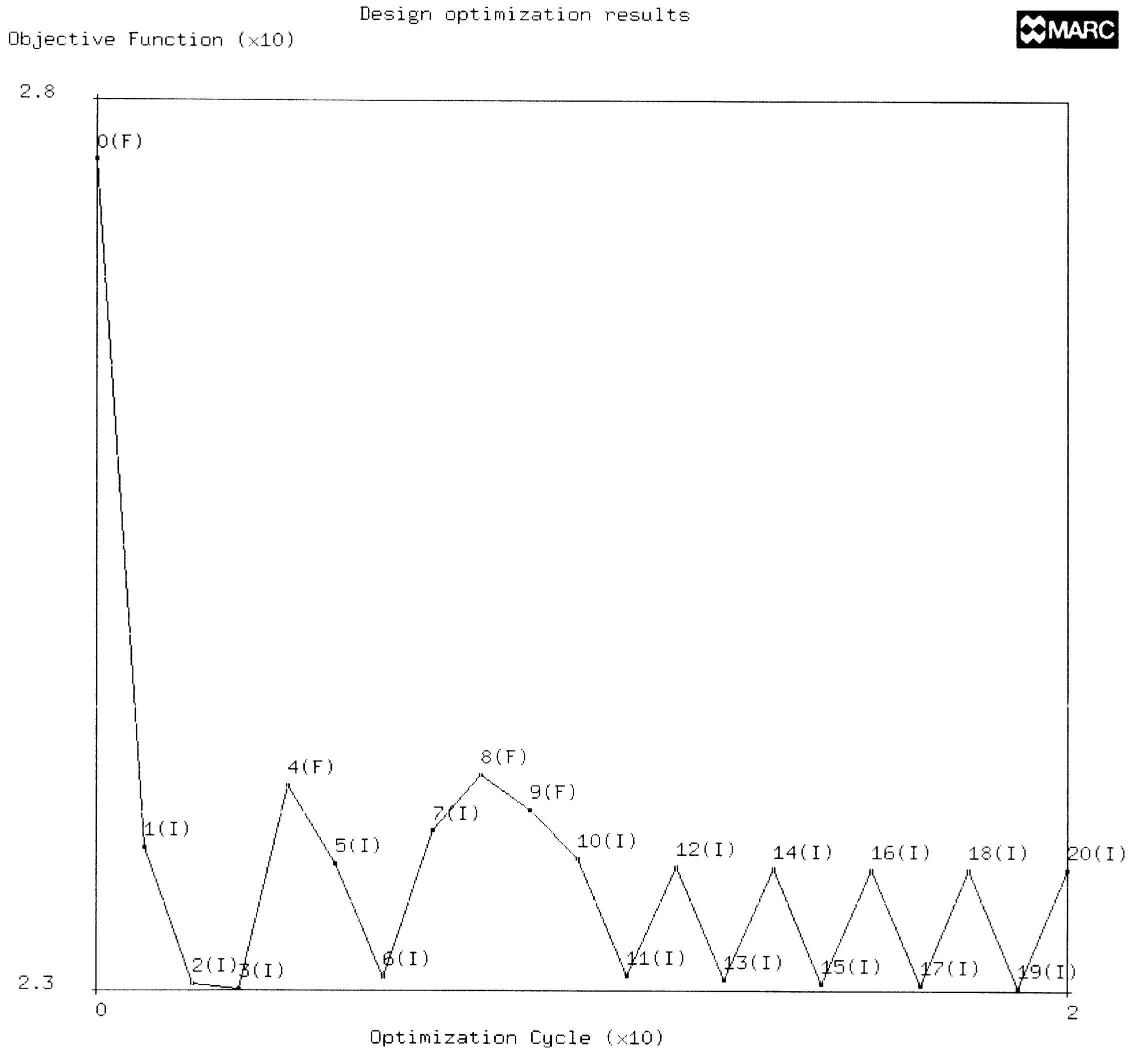

Figure 10.2-4 History Plot of the Objective Function

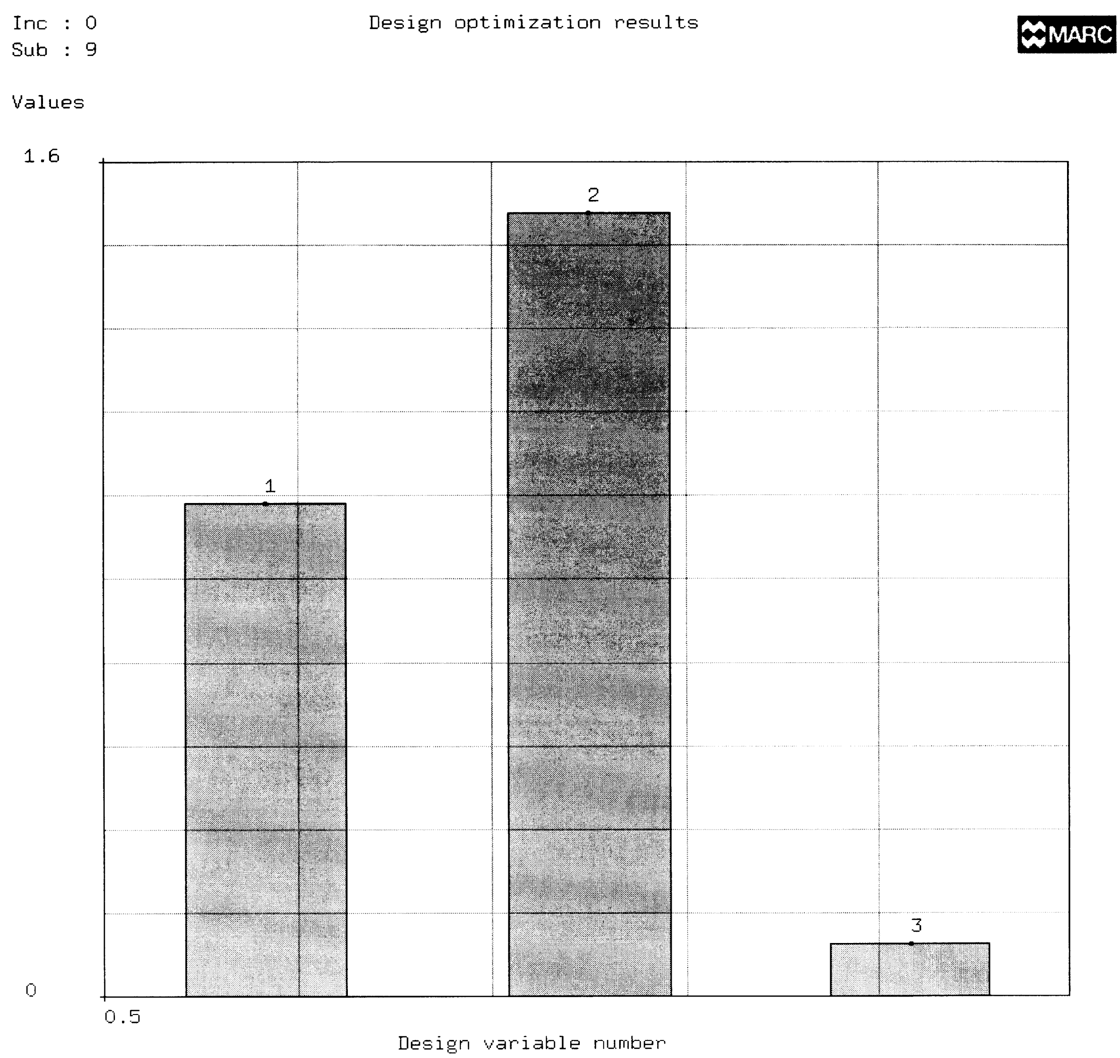

Figure 10.2-5 Design Variables at Best Feasible Design (Cycle 9)

10.3 Design Sensitivity Analysis and Optimization of a Simply-supported Thick Plate

With this problem, we examine the application of the design sensitivity and design optimization procedures for a simply supported thick plate, to be designed for free vibration characteristics and under uniformly distributed pressure.

Elements

Element type 21 is a 20-node isoparametric brick. Eight of the nodes are corner nodes, and twelve are midside nodes. There are three displacement degrees of freedom at each node. Each edge of the brick may be parabolic by means of a curve fitted through the midside node. Numerical integration is accomplished with 27 points using Gaussian quadrature. See Volume B for further details.

Model

Because of symmetry, only one-quarter of the plate is modeled (Figure 10.3-1). One element is used through the thickness, two in each direction in the plane of the plate for a total of four elements. There are 51 nodes and therefore a total of 153 degrees of freedom.

Geometry

No geometry data is used for this element.

Material Properties

The material is isotropic, however, element 4 has two-thirds the mass density of the others.

Loading

The first "load case" consists of an eigenvalue analysis imposed by the MODAL SHAPE data block. As the second load case, a uniform pressure is applied on element 4 by means of the DIST LOADS option. Load type 0 is specified for uniform pressure on the 1-2-3-4 face of element 4.

Boundary Conditions

On the plate edges ($z = 0$, $y = 0$, or $x = 0$, $z = 0$), the plate is simply supported ($w = 0$). On the symmetry planes ($x = 30$ or $y = 30$), in-plane movement is constrained. On the $x = 30$ plane, $u = 0$, and on the $y = 30$ plane, $v = 0$.

Design Variables and Objective Function

There are three design variables for this problem. The first is the Young's modulus for material 1 (elements 1 to 3) with lower and upper limits of 1.8 x 10**7 and 3.0 x 10**7, respectively. The second and the third variables are the mass density and the Possion's ratio, respectively, for the same material. The lower and upper bounds for both of these are 0.1 and 0.4. The objective function for this problem is the total mass of the material used. Thus in design sensitivity analysis we obtain the gradient of the material mass with respect to the design variables. For design optimization, we seek to minimize the total material mass.

Design Constraints

Design constraints are on stress, displacement, and eigenfrequency response. Under the static load case, the maximum absolute valued principal stress and the fifth stress component (shear stress) are constrained for all elements. The translation in the first direction at node 15 is constrained in only one direction. We illustrate a pitfall with the second displacement constraint, which is a relative translation constraint between two nodes 15 and 16, along the second direction. In this case, the actual value is negative, but since absolute value was not specified, the constraint bounded from above becomes irrelevant.

Results

The design sensitivity and design optimization cases are run as separate jobs, with the data files e10x3a.dat and e10x3b.dat, respectively. Figure 10.3-2 shows the gradient of the first eigenfrequency with respect to the design variables. This is a case where the derivatives due to the three variables are all of substantially different orders of magnitude (6.4×10^{-7}; -33.0; 0.24). Thus, the first derivative is shown as 0 on the plot (although, it is important due to the magnitude of E) and the plot of the second is cut off at -1.0. Figure 10.3-3 is a contour plot showing, on the finite element model, the nodal averaged element contributions to the response quantity in question. Figure 10.3-4 shows the change in the objective function with the optimization cycle in the form of a history plot. The last cycle (cycle 7) is seen to give the best feasible (F) design. Figure 10.3-5 shows the change in the third design variable (Poisson's ratio) during optimization.

Parameters, Options, and Subroutines Summary

Listed below are the options used in example e10x3a.dat:

Parameters	Model Definition Options	History Definition Options
DESIGN SENSITIVITY	CONNECTIVITY	CONTINUE
DYNAMIC	COORDINATES	DIST LOADS
ELEMENTS	DESIGN DISPLACEMENT CONSTRAINTS	MODAL SHAPE
END	DESIGN FREQUENCY CONSTRAINTS	
SIZING	DESIGN OBJECTIVE	
TITLE	DESIGN STRESS CONSTRAINTS	
	DESIGN VARIABLES	
	DIST LOADS (dummy)	
	END OPTION	
	FIXED DISP	
	ISOTROPIC	
	POST	

Listed below are the options used in example e10x3b.dat:

Parameters	Model Definition Options	History Definition Options
DESIGN OPTIMIZATION	CONNECTIVITY	CONTINUE
DYNAMIC	COORDINATES	DIST LOADS
ELEMENTS	DESIGN DISPLACEMENT CONSTRAINTS	MODAL SHAPE
END	DESIGN FREQUENCY CONSTRAINTS	
SIZING	DESIGN OBJECTIVE	
TITLE	DESIGN STRESS CONSTRAINTS	
	DESIGN VARIABLES	
	DIST LOADS (dummy)	
	END OPTION	
	FIXED DISP	
	ISOTROPIC	
	POST	

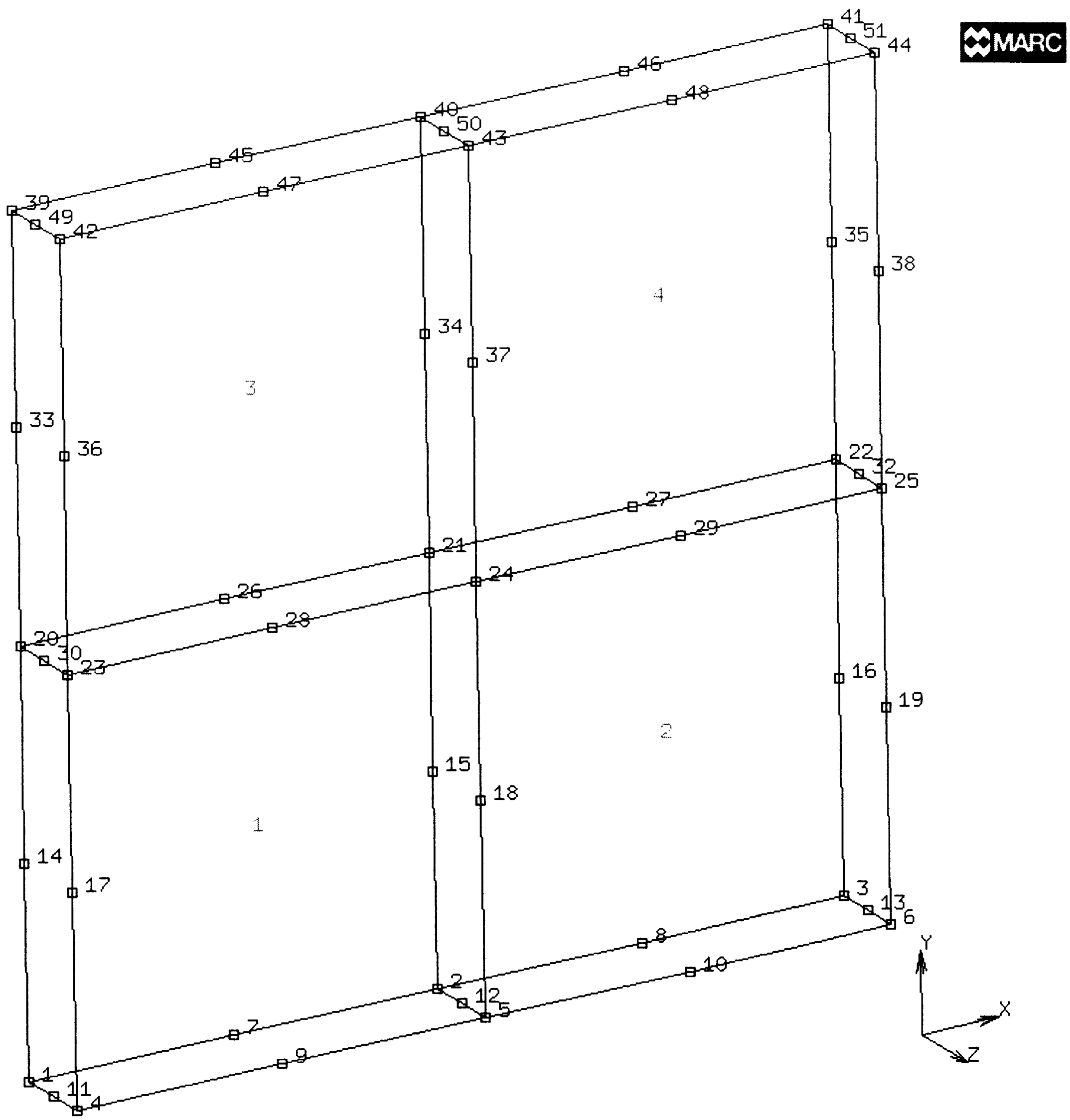

Figure 10.3-1 Finite Element Model of Quarter Thick Plate

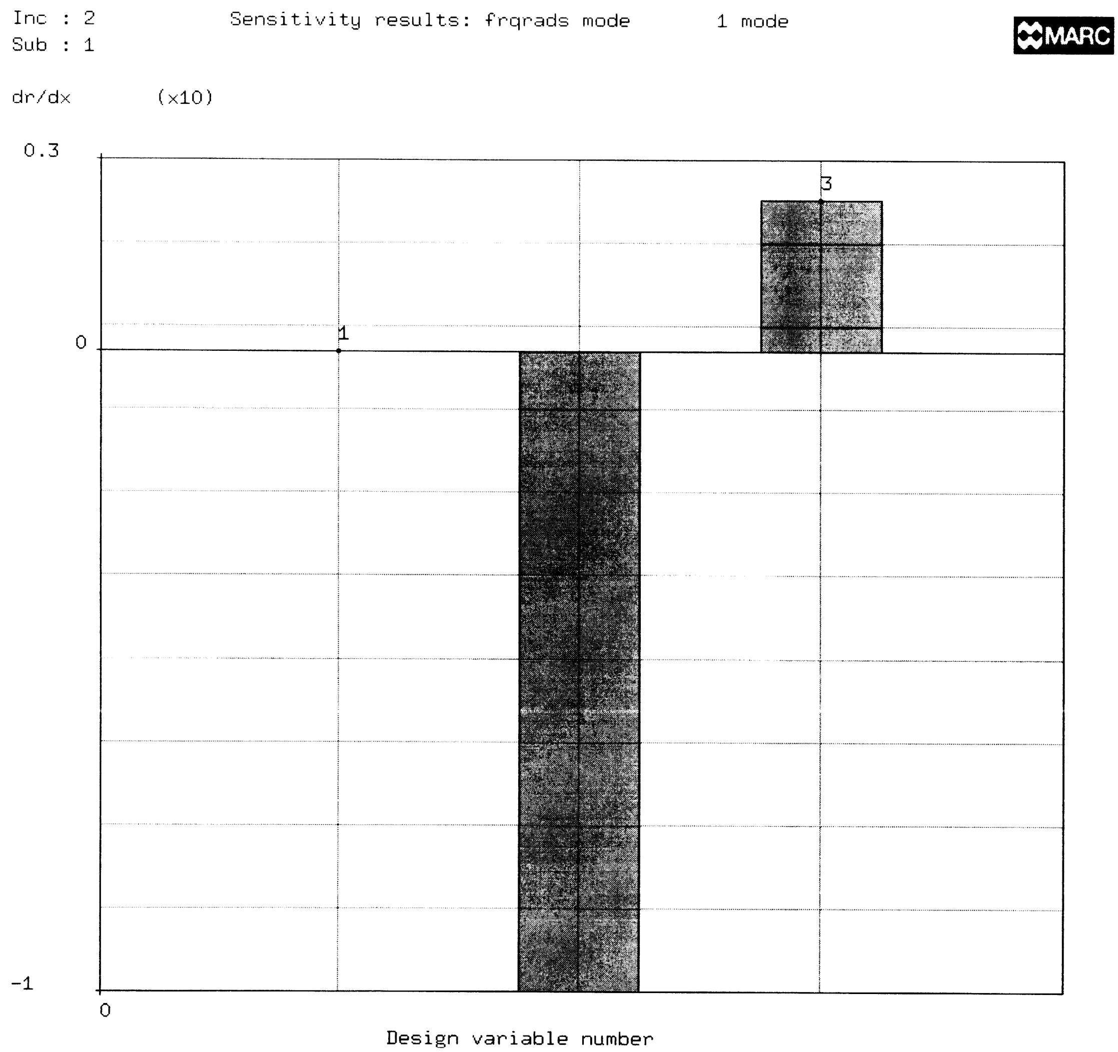

Figure 10.3-2 Gradient of the First Eigenfrequency

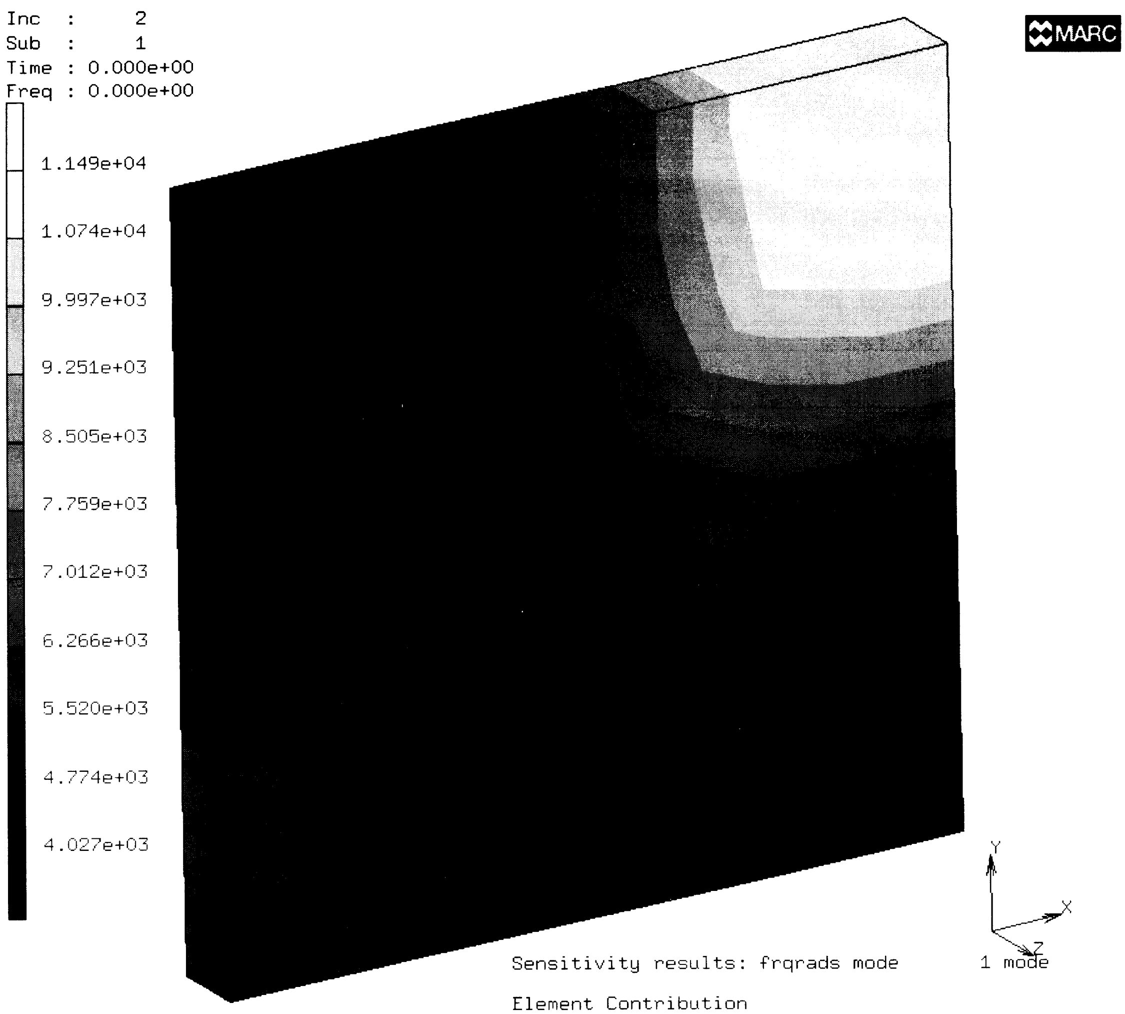

Figure 10.3-3 Element Contributions to the First Eigenfrequency

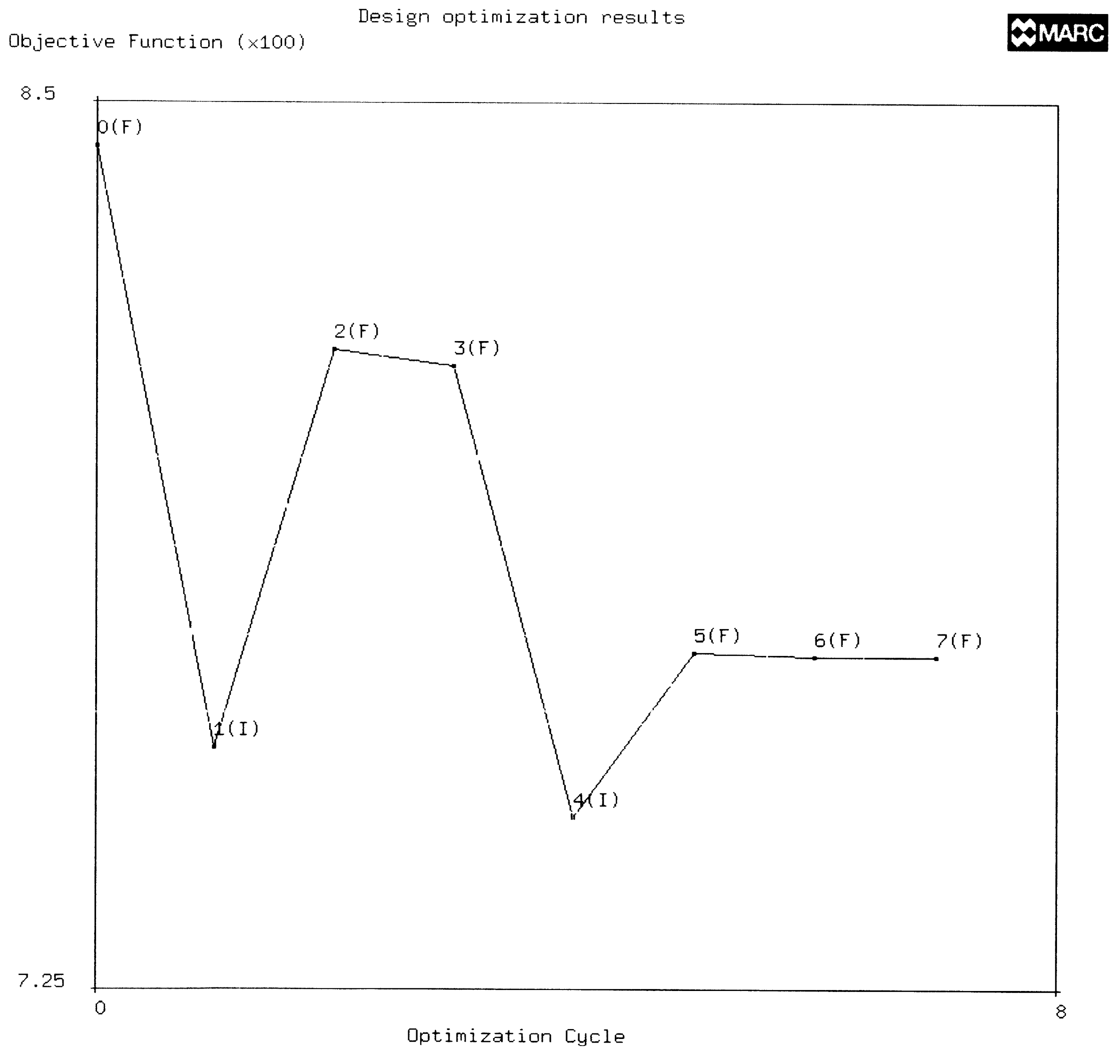

Figure 10.3-4 History Plot of the Objective Function

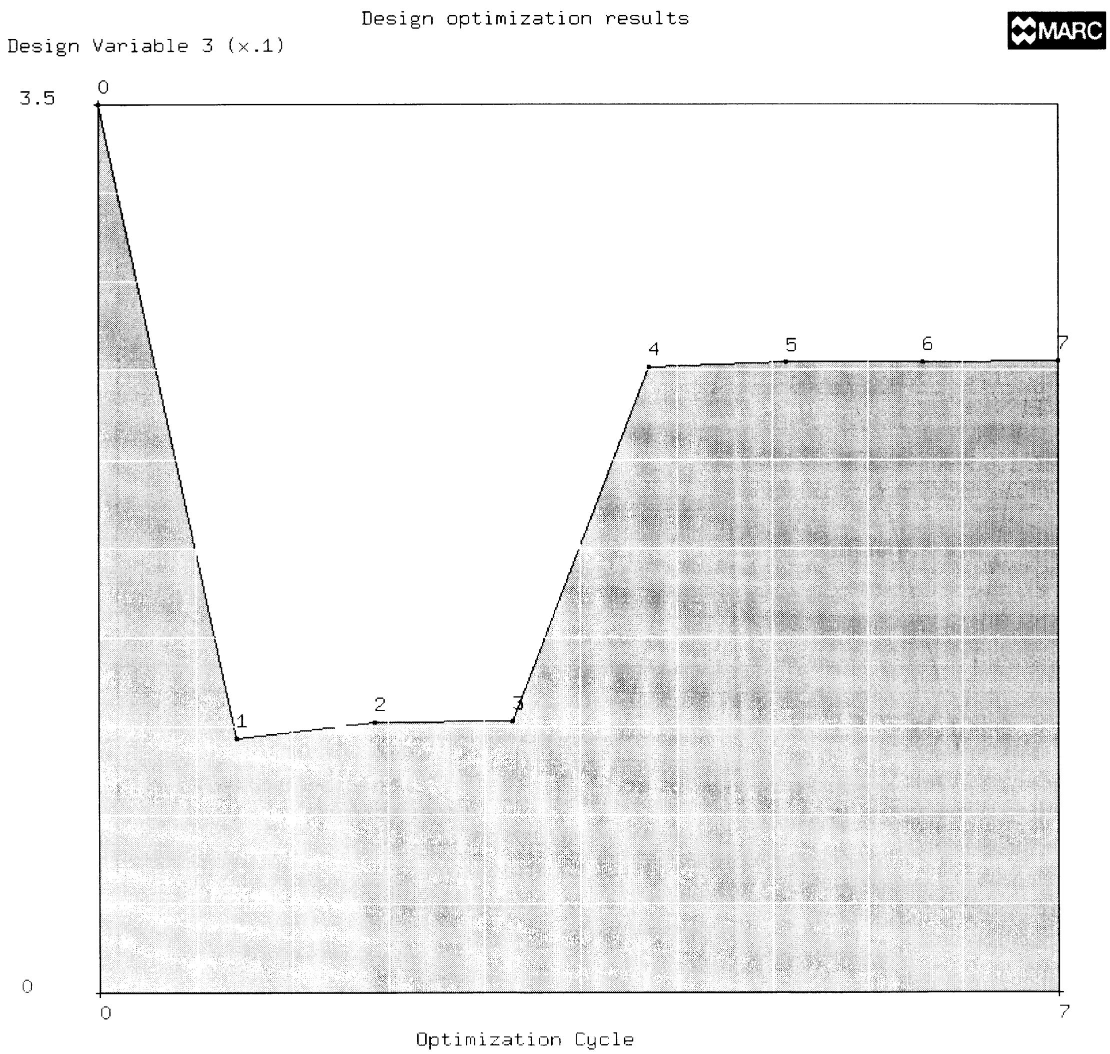

Figure 10.3-5 History Plot of the Poisson's Ratio

10.4 Design Sensitivity Analysis and Optimization of a Shell Roof

A shell type roof structure is considered for design sensitivity analysis and design optimization under the action of a static load case and for eigenfrequency response.

Elements

Element type 75 is a 4-node, thick-shell element with six global degrees of freedom per node.

Model

The finite element model is shown in Figure 10.4-1. The roof is modeled with 64 type 75 shell elements resulting in a total of 81 nodes.

Geometry

For sensitivity analysis purposes, a thickness of 0.01 is specified for elements from 1 through 54, and 0.015 is specified for elements from 55 through 64, using the GEOMETRY option.

Material Properties

The material is linearly elastic with a Young's modulus of 100,000 and a Poisson's ratio of 0.3.

Loading

The first load case consists of an eigenvalue analysis for free vibration. For the second, and static, load case, a point load and a moment resultant are applied at node 1 by means of the POINT LOAD option.

Boundary Conditions

Various boundary conditions are applied along the four edges.

Design Variables and Objective Function

The two of design variables for this problem are:

1. the thickness of elements 1 through 54
2. the thickness of elements 55 through 64

The objective function for the problem is the total mass of the material used, which means that we seek to minimize the mass by way of the design optimization procedure.

Design Constraints

The design constraints consist of stress constraints and frequency constraints. Stress constraints are imposed for certain elements on the generalized stresses and on stress components. Frequency constraints are on the fundamental frequency and on the difference between the frequencies of the first two modes.

Results

The design sensitivity and design optimization cases are run as separate jobs, with the data files e10x4a.dat and e10x4b.dat, respectively. Figure 10.4-2 shows the gradient of the difference between the first and second eigenfrequencies with respect to the two design variables. This difference is obviously governed by the first design variable; since the derivative due to the second variable is two orders of magnitude smaller, while the two variables are of the same order of magnitude. Figure 10.4-3 is a contour plot showing, on the finite element model, the element contributions to the response quantity in question. Figure 10.4-4 shows the change in the objective function with the optimization cycle in the form of a history plot. The best feasible (F) design is obtained at cycle 18. Figure 10.4-5 is a bar chart showing the values of the design variables at the best feasible design obtained.

Parameters, Options, and Subroutines Summary

Listed below are the options used in example e10x4a.dat:

Parameters	Model Definition Options	History Definition Options
ALL POINTS	CONNECTIVITY	CONTINUE
DESIGN SENSITIVITY	COORDINATES	MODAL SHAPE
DYNAMIC	DESIGN FREQUENCY CONTRAINTS	POINT LOAD
ELEMENTS	DESIGN OBJECTIVE	
END	DESIGN STRESS CONSTRAINTS	
SETNAME	DESIGN VARIABLES	
SHELL SECT	END OPTION	
SIZING	FIXED DISP	
TITLE	GEOMETRY	
	ISOTROPIC	
	OPTIMIZE	
	POINT LOAD (dummy)	
	POST	
	SOLVER	

Listed below are the options used in example e10x4b.dat:

Parameters	Model Definition Options	History Definition Options
ALL POINTS	CONNECTIVITY	CONTINUE
DESIGN OPTIMIZATION	COORDINATES	MODAL SHAPE
DYNAMIC	DESIGN FREQUENCY CONTRAINTS	POINT LOAD
ELEMENTS	DESIGN OBJECTIVE	
END	DESIGN STRESS CONSTRAINTS	
SETNAME	DESIGN VARIABLES	
SHELL SECT	END OPTION	
SIZING	FIXED DISP	
TITLE	GEOMETRY	
	ISOTROPIC	
	OPTIMIZE	
	POINT LOAD (dummy)	
	POST	
	SOLVER	

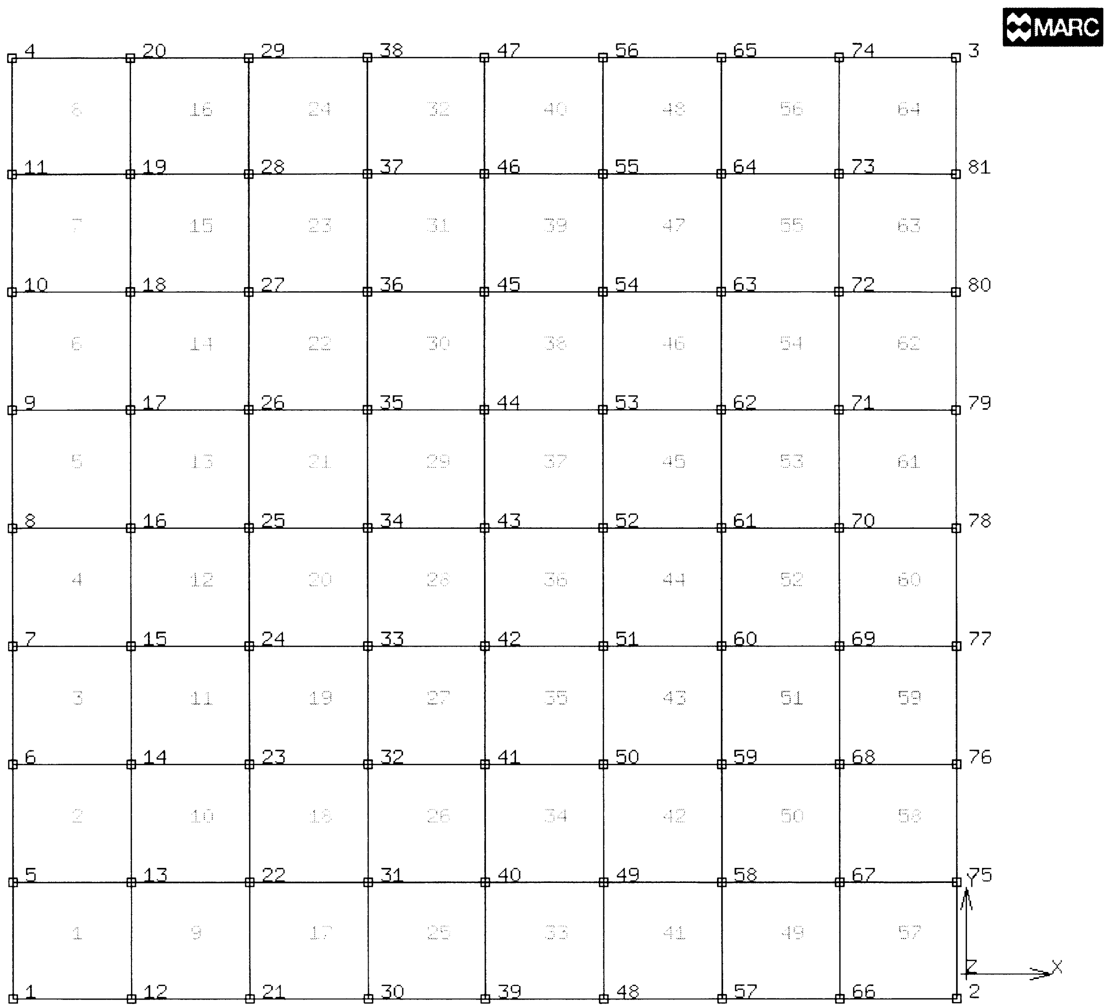

Figure 10.4-1 Finite Element Model of the Shell Roof

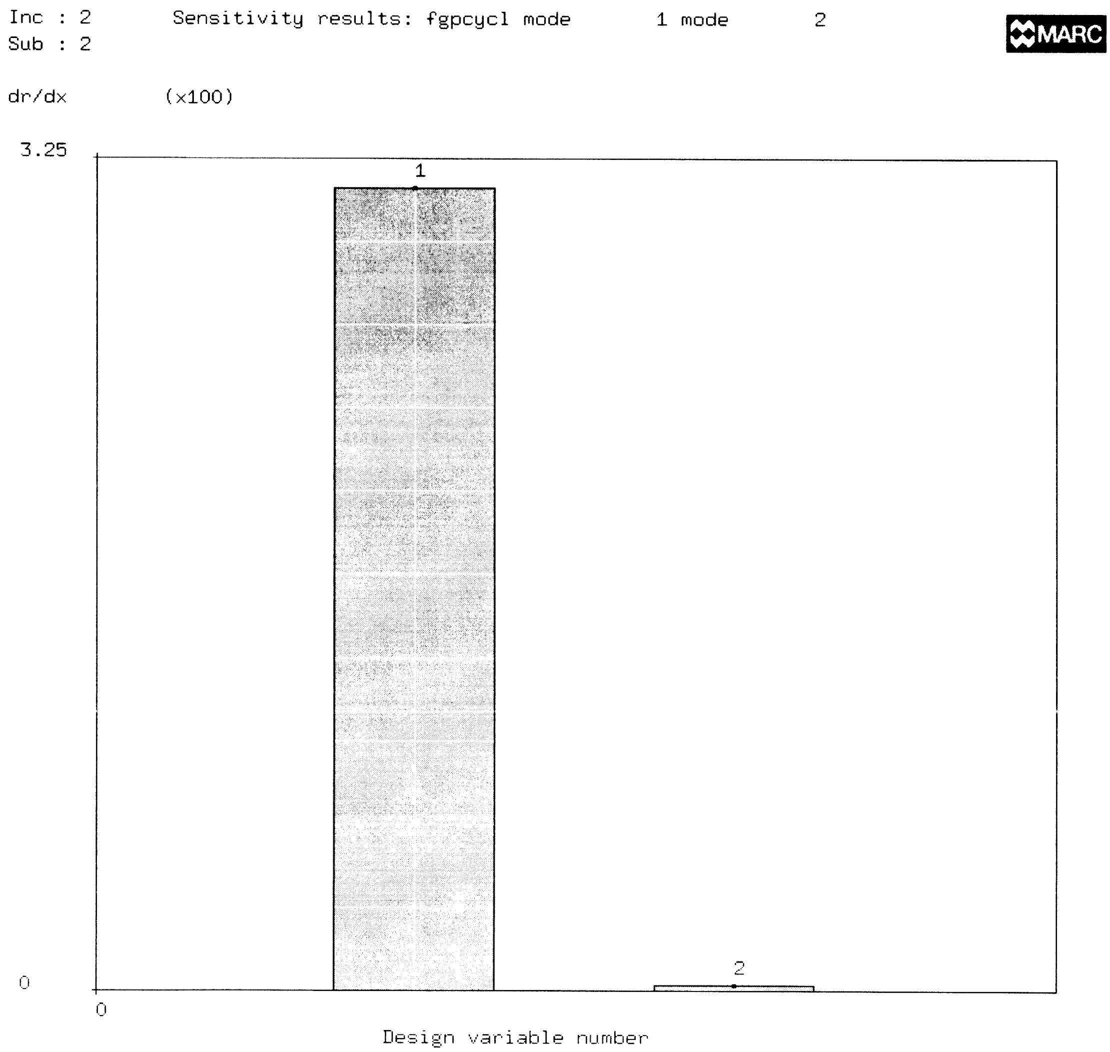

Figure 10.4-2 Gradient of Difference Between First and Second Eigenfrequencies

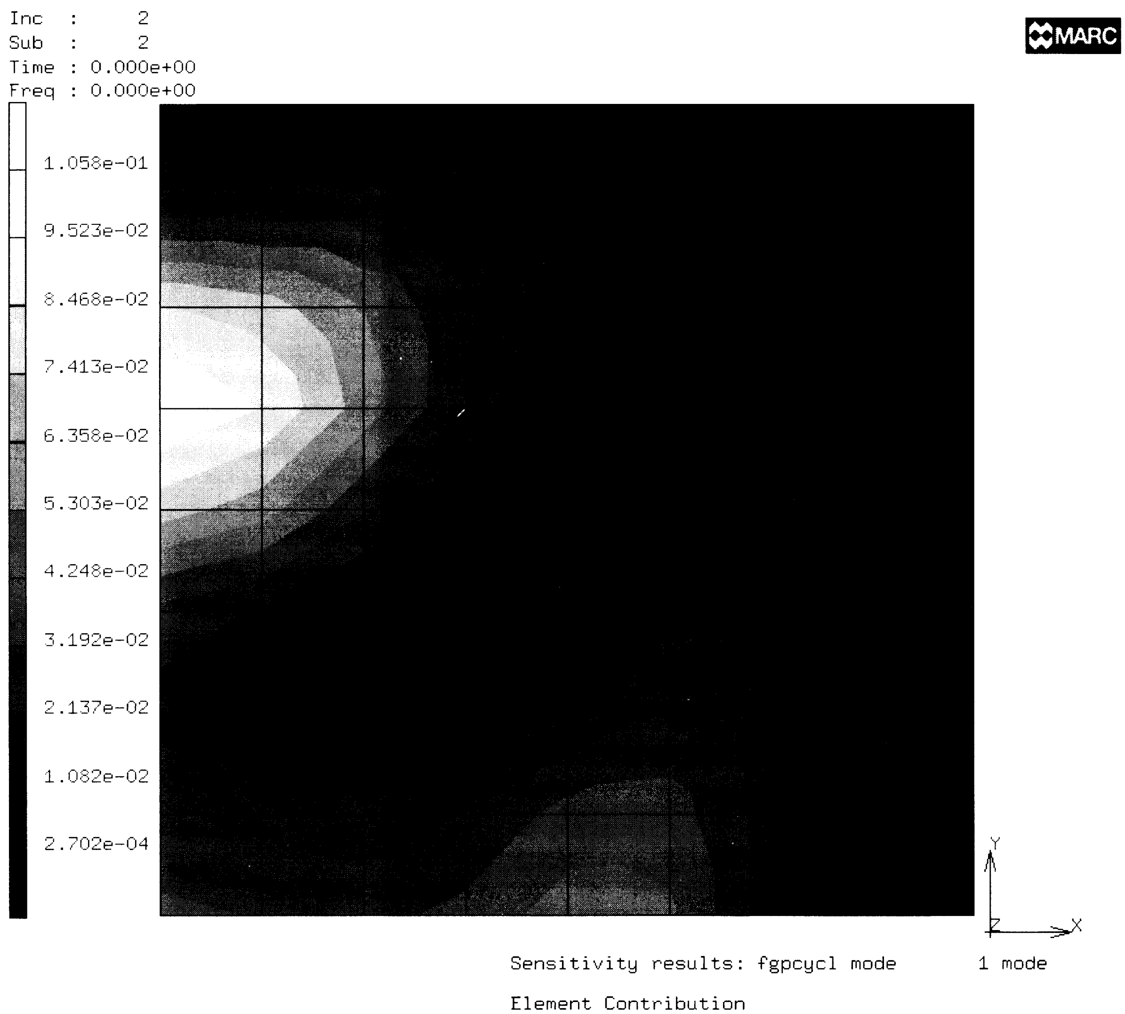

Figure 10.4-3 Element Contributions to the Response of Figure 10.4-2

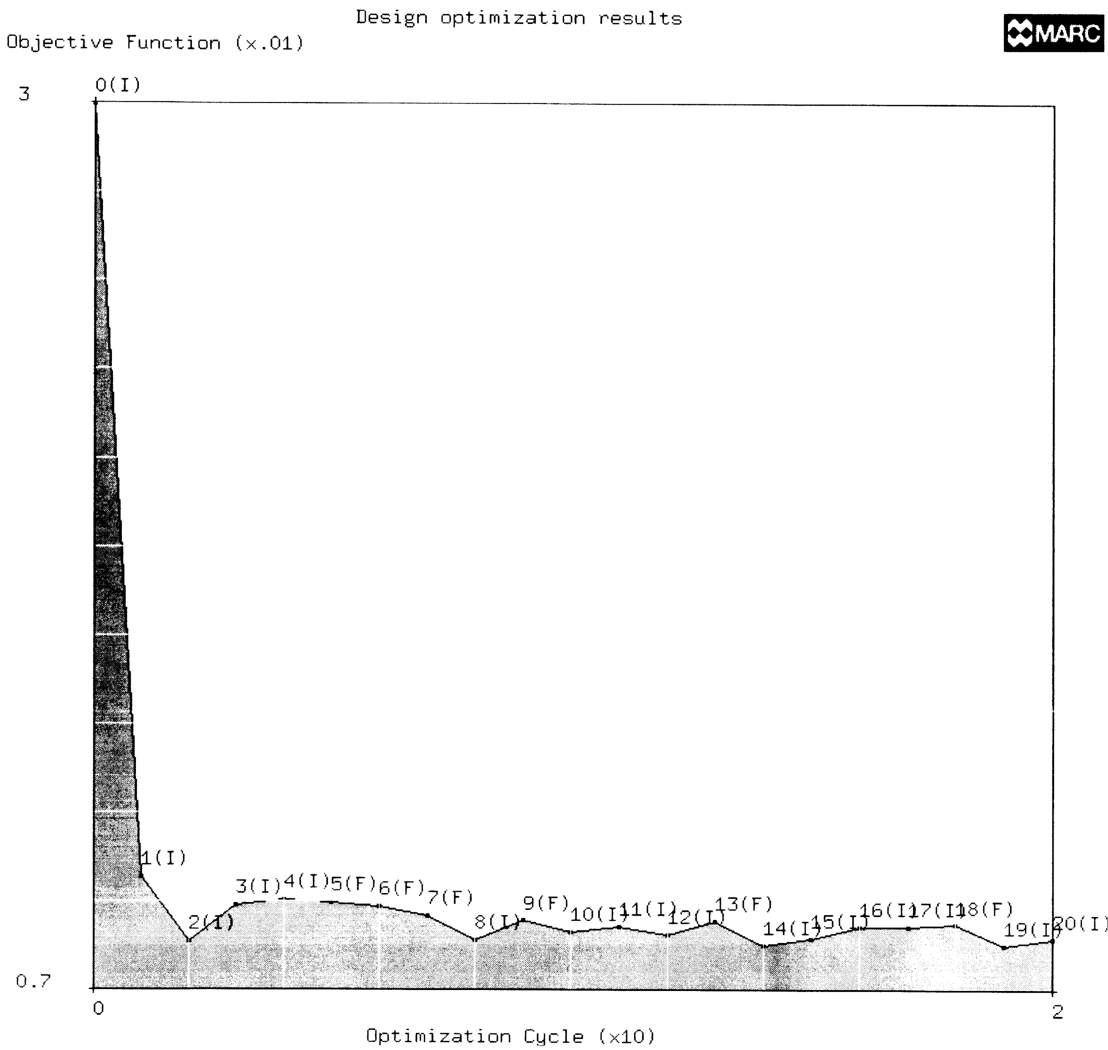

Figure 10.4-4 History Plot of the Objective Function

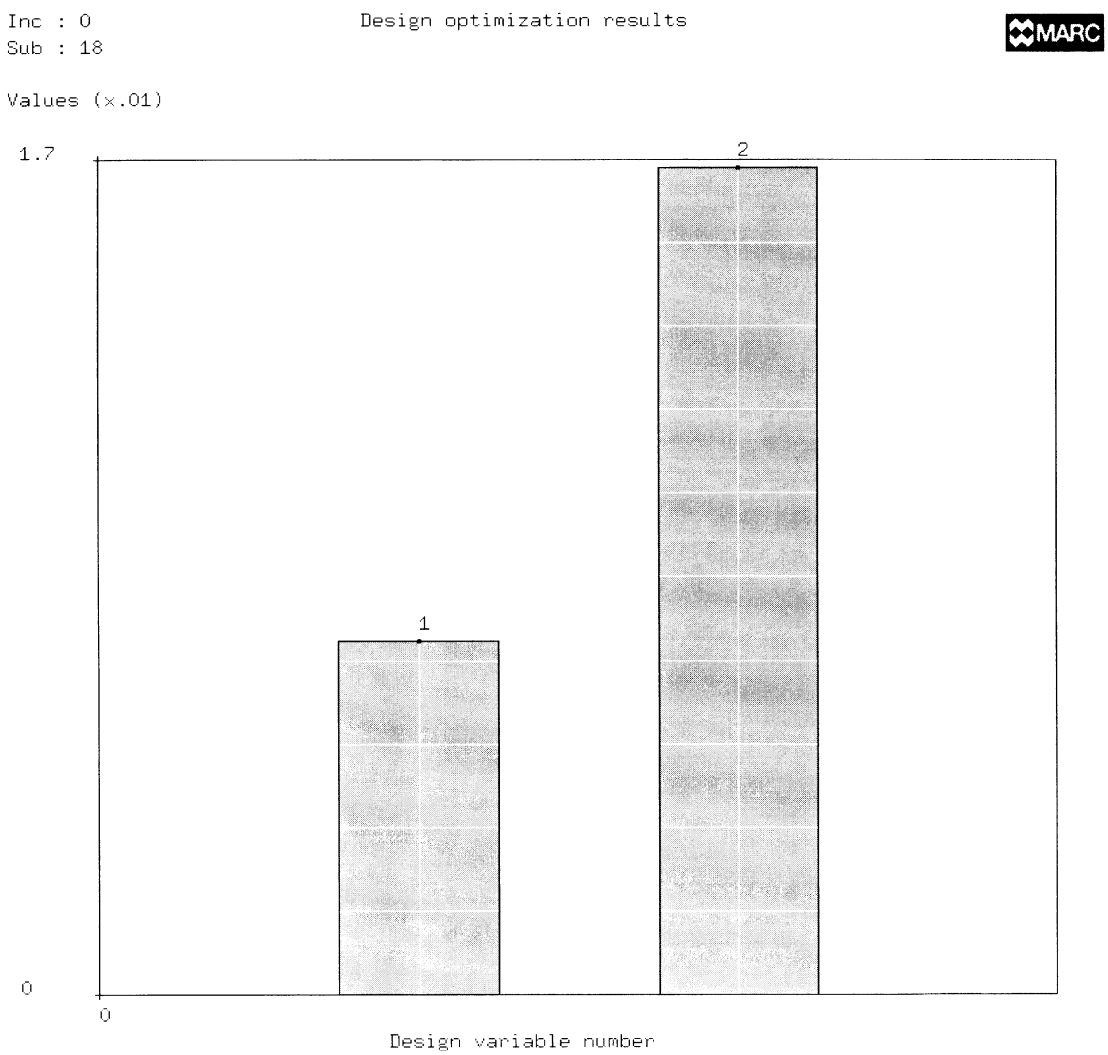

Figure 10.4-5 Design Variables at Best Feasible Design (Cycle 18)

10.5 Design Sensitivity Analysis and Optimization of a Composite Plate

This problem demonstrates the utilization of the design sensitivity and optimization procedures for a rectangular plate made up of multilayered composite material.

Element)

Element type 75 is a 4-node, thick-shell element with six global degrees of freedom per node.

Model

The model is shown in Figure 10.5-1. The plate is modeled with six type 75 elements and a total of 14 nodes.

Geometry

The elements are modeled as composites with nine layers. For sensitivity analysis, the prescribed layer thicknesses are 5.166 cm for layers 1 through 6, 0.272 cm for layer 7, and 3.364 cm for layers 8 and 9. The ply angle is zero degrees for all layers.

Material Properties

The composite elements contain two material types, one is an orthotropic material which is used for layer 7, and the other is an isotropic material used for the rest of the layers.

Loading and Boundary Conditions

The single load case consists of point loads of 350 applied through the POINT LOAD option to nodes 9 and 10 in the negative third direction. Various appropriate boundary conditions are applied.

Design Variables and Objective Function

The two types of design variables chosen are the ply angle and the layer thickness. The ply angle at layer 7 is the first design variable. The second and third design variables are the layer thicknesses linked over layers 1 through 6 and 8 and 9, respectively. The ply angle variable can change between 0.1 and 180 degrees. The lower and upper bounds for the layer thickness variables are 0.1 to 8.0 cm and 0.1 to 5.0 cm respectively.

The objective function for this problem is the total volume of the material used. For design sensitivity, we request the gradient of the total material volume. For design optimization, we seek to minimize the total material volume.

Design Constraints

Design constraints are imposed on stress, displacement, and strain response quantities. Stress constraints, which are on the von Mises stress, generalized stresses, and a normal stress component, are imposed for element 6 only. Displacement constraints consist of a bound on the translation in the second direction for node 6, and a limit on the relative translation in the second direction between nodes 4 and 5. The single strain constraint sets a limit on the magnitude of the second normal strain component.

Results

The design sensitivity and design optimization cases are run as separate jobs, with the data files e10x5a.dat and e10x5b.dat, respectively. For this particular case, the only difference between the two files is the parameter line specifying DESIGN SENSITIVITY or DESIGN OPTIMIZATION. Figure 10.5-2 shows the gradient of the relative y-direction translation between nodes 4 and 5 with respect to the design variables at the user prescribed design. Figure 10.5-3 is a contour plot showing, on the finite element model, the element contributions to the response quantity in question. Figure 10.5-4 shows the change in the objective function with the optimization cycle in the form of a history plot. The best feasible (F) design within twenty cycles is obtained at cycle 8. Figure 10.5-5 is a bar chart showing the values of the design variables at the best design obtained.

Parameters, Options, and Subroutines Summary

Listed below are the options used in example e10x5a.dat:

Parameter Options	Model Definition Options	History Definition Options
DESIGN SENSITIVITY	COMPOSITE	CONTINUE
DYNAMIC	CONN GENER	POINT LOAD
ELEMENTS	CONNECTIVITY	
END	COORDINATES	
PRINT	DESIGN DISPLACEMENTS	
SIZING	DESIGN OBJECTIVE	
TITLE	DESIGN STRAIN CONSTRAINTS	
	DESIGN STRESS CONSTRAINTS	
	DESIGN VARIABLES	
	END OPTION	
	FIXED DISP	
	ISOTROPIC	
	NODE FILL	
	ORIENTATION	
	ORTHOTROPIC	

Listed below are the options used in example e10x5b.dat:

Parameter Options	**Model Definition Options**	**History Definition Options**
DESIGN OPTIMIZATION	COMPOSITE	CONTINUE
DYNAMIC	CONN GENER	POINT LOAD
ELEMENTS	CONNECTIVITY	
END	COORDINATES	
PRINT	DESIGN DISPLACEMENTS CONSTRAINTS	
SIZING	DESIGN OBJECTIVE	
TITLE	DESIGN STRAIN CONSTRAINTS	
	DESIGN STRESS CONSTRAINTS	
	DESIGN VARIABLES	
	END OPTION	
	FIXED DISP	
	ISOTROPIC	
	NODE FILL	
	ORIENTATION	
	ORTHOTROPIC	

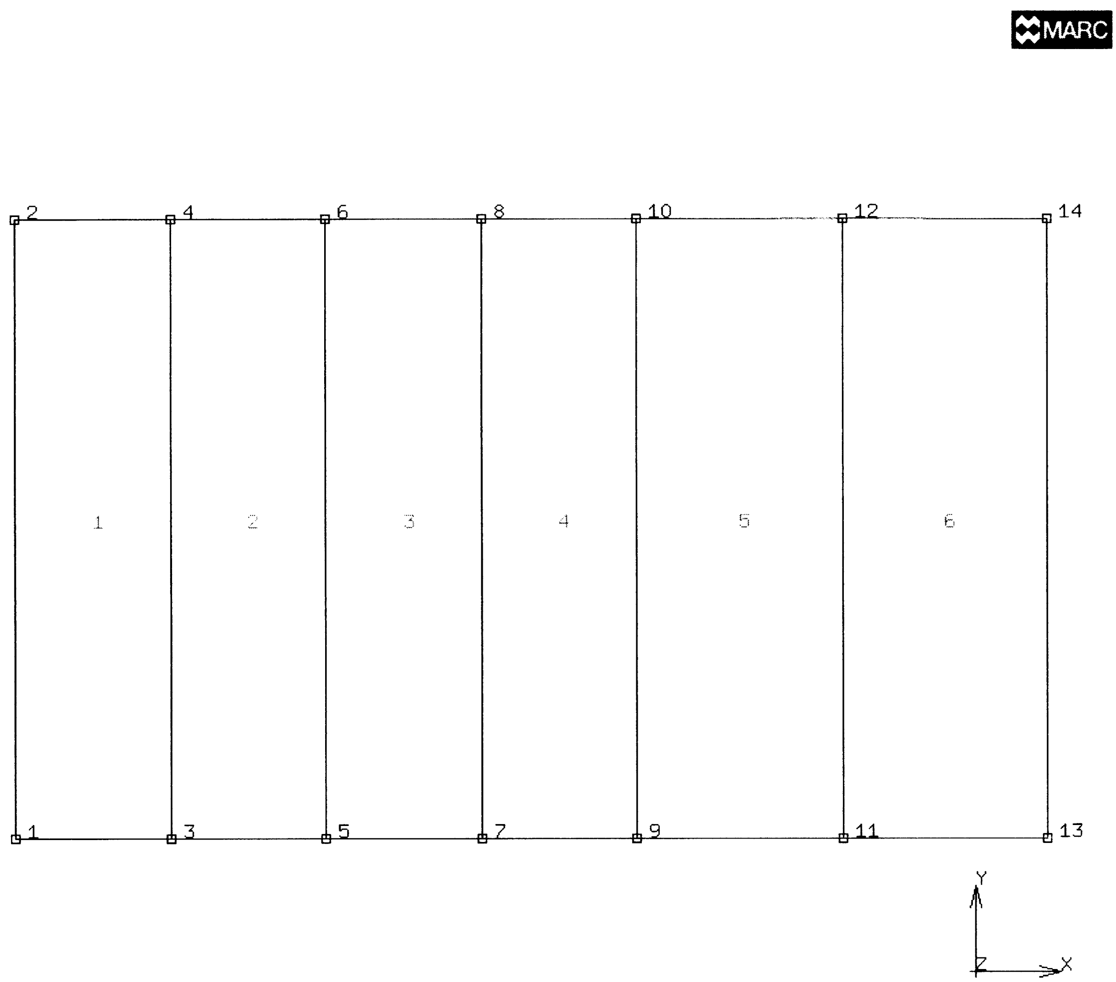

Figure 10.5-1 Finite Element Model of Composite Plate

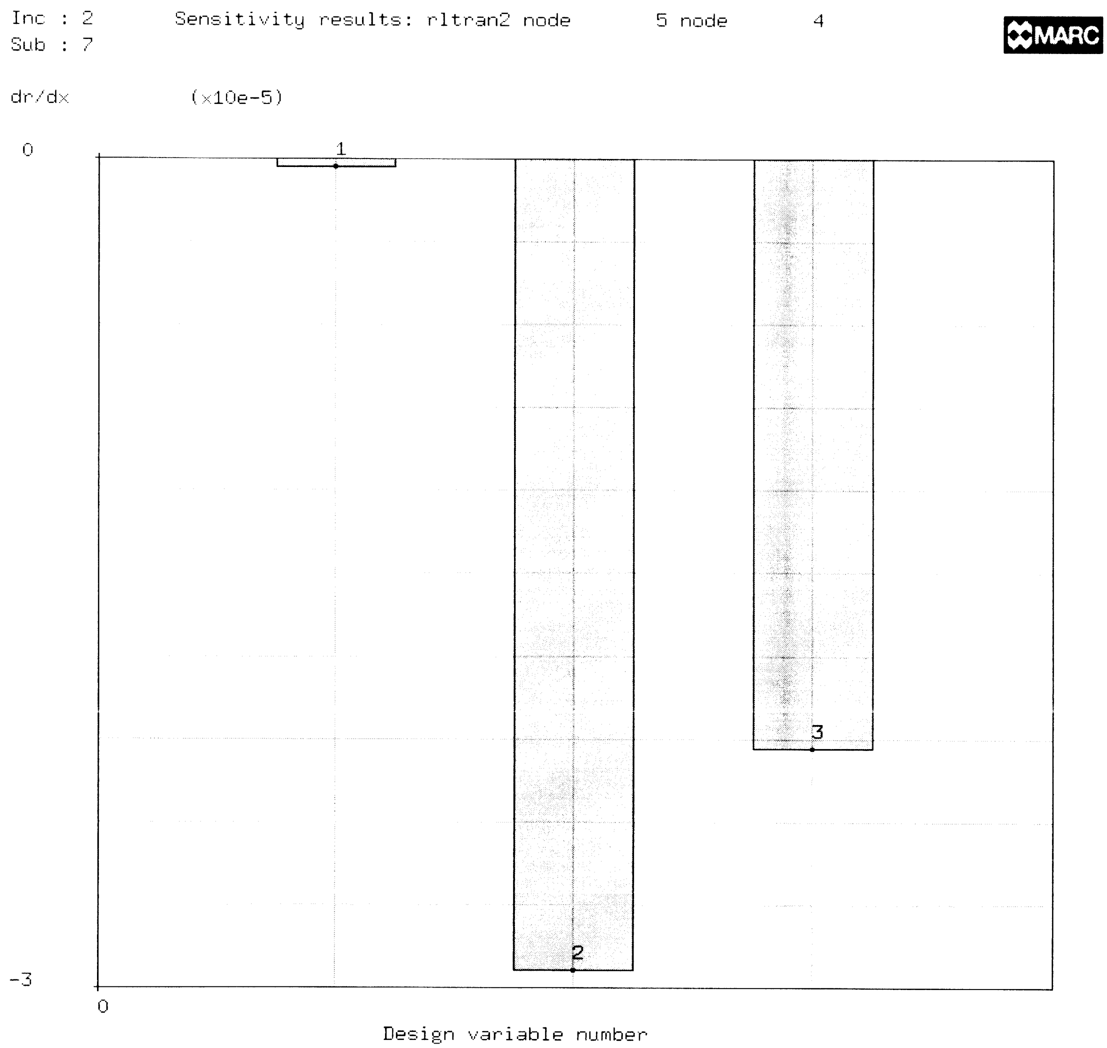

Figure 10.5-2 Gradient of Relative y-direction Translation Between Nodes 4 and 5

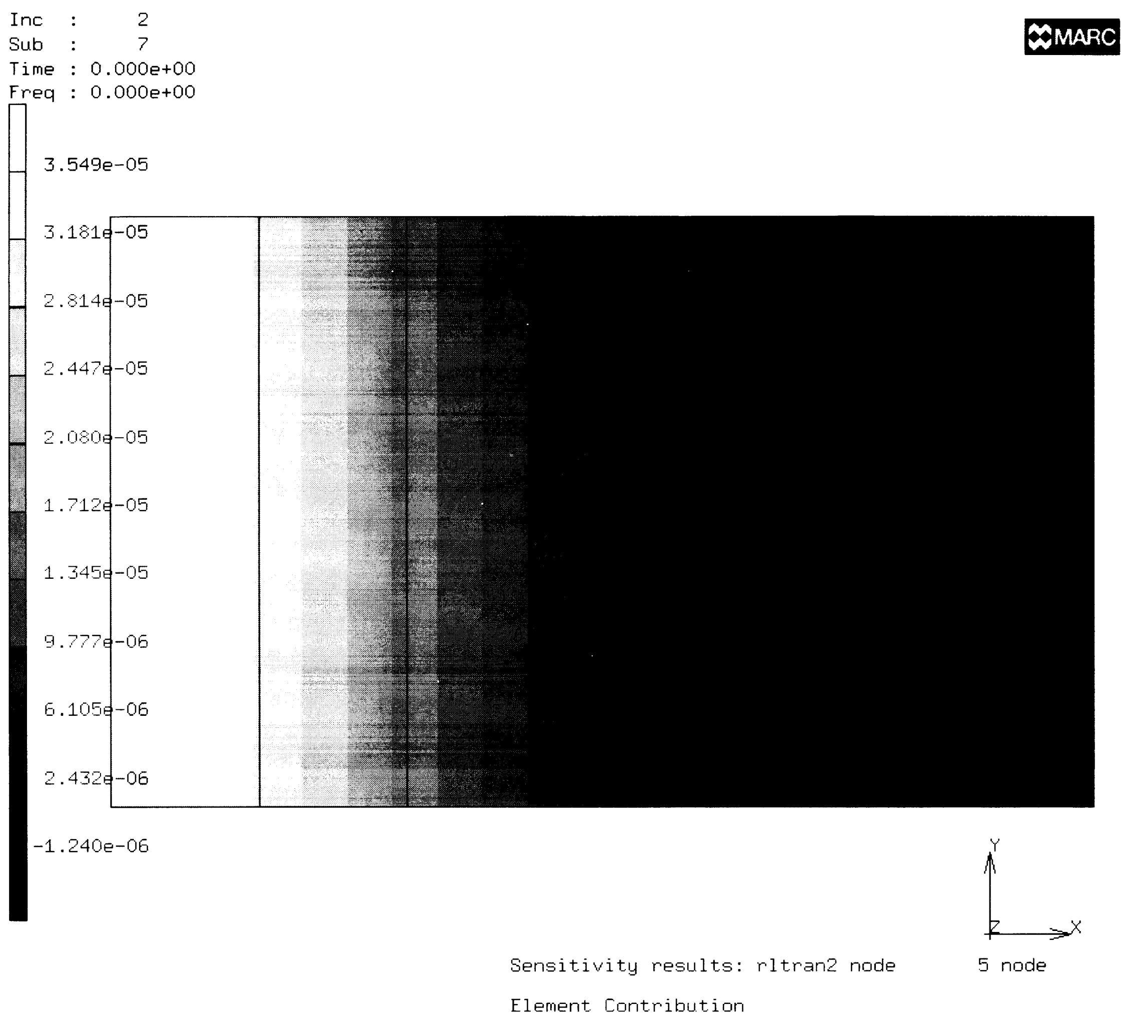

Figure 10.5-3 Element Contributions to Response of Figure 10.5-2

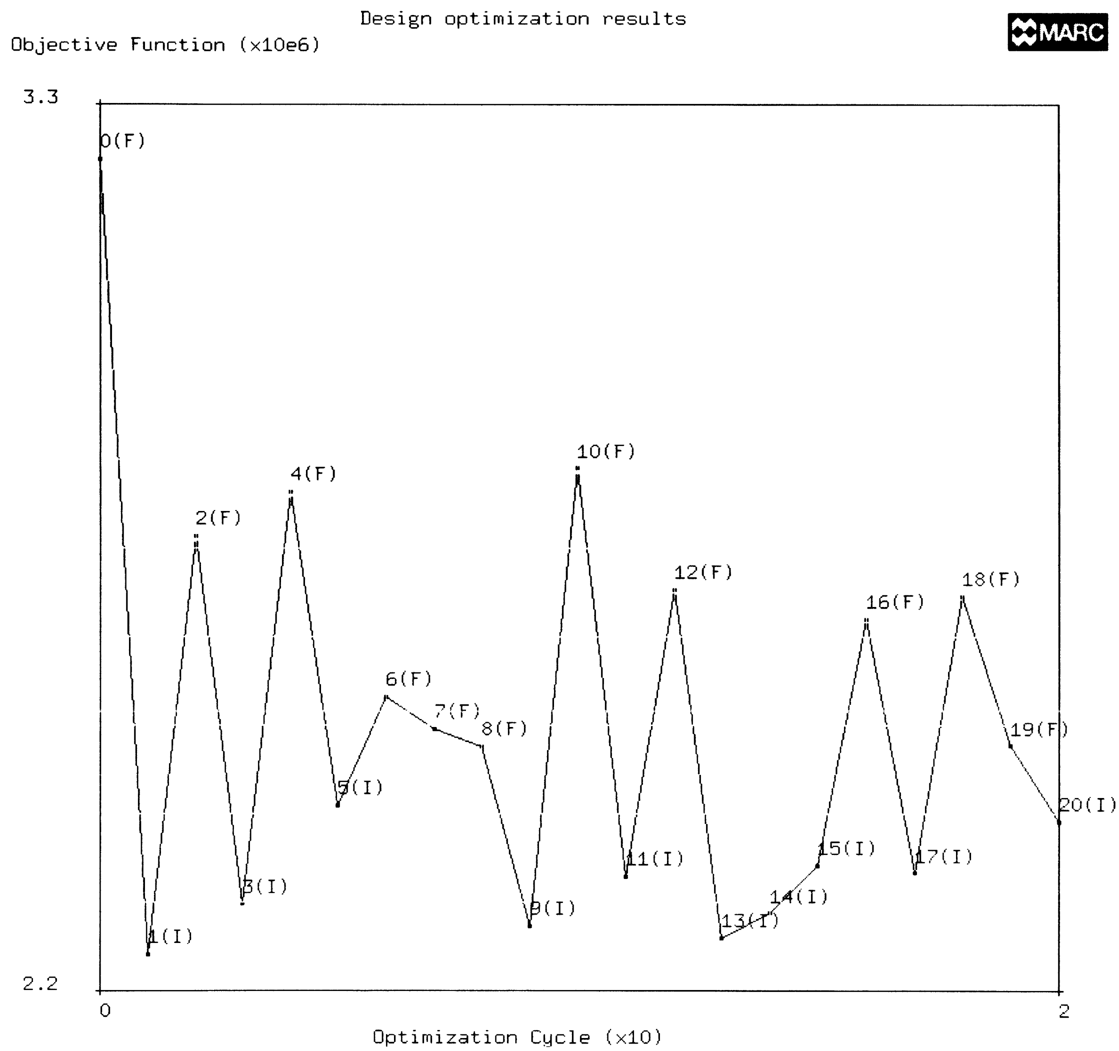

Figure 10.5-4 History Plot of the Objective Function

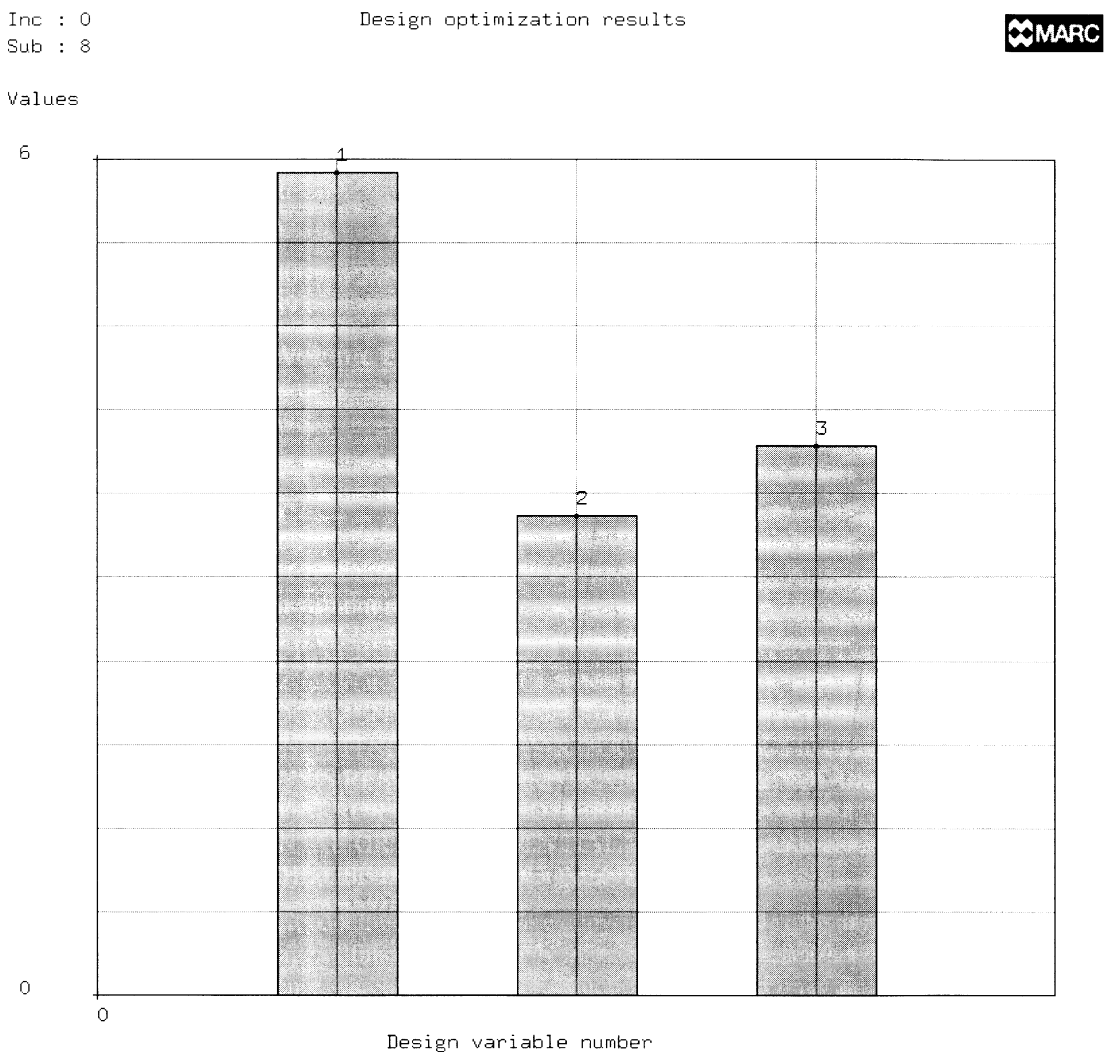

Figure 10.5-5 Design Variables at Best Feasible Design (Cycle 8)

10.6 Design Sensitivity Analysis and Optimization of a Ten-bar Truss

The cantilever ten-bar truss of Figure 10.6-1 is subjected to a single static load case. Design sensitivity analysis and design optimization are conducted with constraints on the stresses in the truss elements.

Elements

Element type 9 is a 2-node, 3D straight truss bar element with constant cross section and three degrees of freedom per node.

Model

The model is shown in Figure 10.6-1. It has a total of 10 axial bar elements and 6 nodes. The two nodes at the left end (nodes 2 and 4) are pinned to prevent any translation at these nodes. The total length of the cantilever is 20 units with a height of 10 units.

Geometry

The cross-sectional areas are five units each for purposes of design sensitivity analysis. The limits for design optimization are given with the DESIGN VARIABLES option.

Material Properties

The material is linearly elastic with a Young's modulus of 10,000 and a mass density of 1.0.

Loading

Point loads of 50 and 100 are applied to nodes 1 and 5, respectively, along the positive second degree of freedom (that is, upwards in Figure 10.6-1) through the POINT LOAD option.

Design Variables and Objective Function

For this problem, the only type of design variable is the cross-sectional areas. These design variables are unlinked over the elements, resulting in ten independent design variables, each corresponding to the cross-sectional area of a given truss element. Material mass is the prescribed objective function.

Design Constraints

The design constraints are on the axial stresses in the truss bars. The limit on the stresses is the same for both tension and compression, although MARC allows the specification of different limits.

Results

The design sensitivity and design optimization cases are run as separate jobs, with the data files e10x6a.dat and e10x6b.dat, respectively. Figure 10.6-2 shows the gradient of the axial stress in element 4 with respect to all ten design variables. Figure 10.6-3 shows the element contributions to the response quantity in question. Figure 10.6-4 shows the change in the objective function with the optimization cycle in the form of a history plot. Cycle 50 is seen to give the best feasible (F) design. Figure 10.6-5 is a bar chart showing the values of the design variables at the best feasible design obtained. It will be noted that the elements carrying the highest loads, elements 1 and 10, are the ones ending up with the largest cross-sectional areas in this stress-constrained problem.

Parameters, Options, and Subroutines Summary

Listed below are the options used in example e10x6a.dat:

Parameters	Model Definition Options	History Definition Options
ALL POINTS	CONNECTIVITY	CONTINUE
DESIGN SENSITIVITY	COORDINATES	POINT LOAD
ELEMENTS	DESIGN OBJECTIVE	
END	DESIGN STRESS CONSTRAINTS	
SETNAME	DESIGN VARIABLES	
SIZING	END OPTION	
TITLE	FIXED DISP	
	GEOMETRY	
	ISOTROPIC	
	NO PRINT	
	OPTIMIZE	
	POST	
	SOLVER	

Listed below are the options used in example e10x6b.dat:

Parameters	**Model Definition Options**	**History Definition Options**
ALL POINTS	CONNECTIVITY	CONTINUE
DESIGN OPTIMIZATION	COORDINATES	POINT LOAD
ELEMENTS	DESIGN OBJECTIVE	
END	DESIGN STRESS CONSTRAINTS	
SETNAME	DESIGN VARIABLES	
SIZING	END OPTION	
TITLE	FIXED DISP	
	GEOMETRY	
	ISOTROPIC	
	NO PRINT	
	OPTIMIZE	
	POST	
	SOLVER	

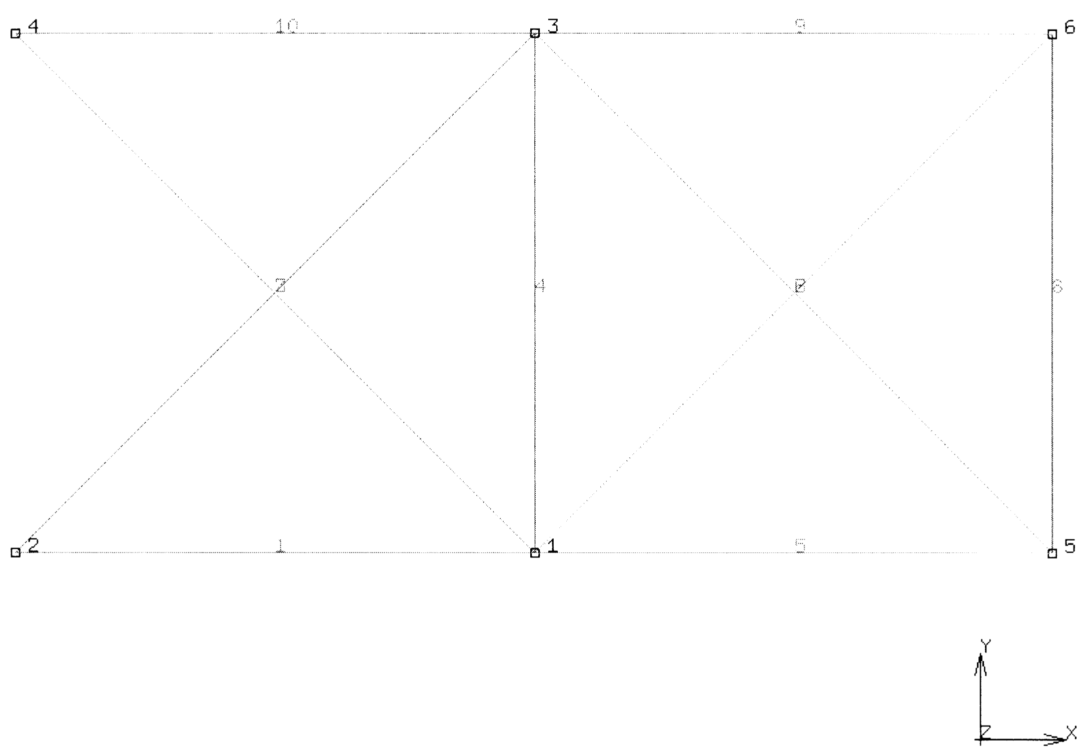

Figure 10.6-1 Finite Element Model of Ten-bar Truss

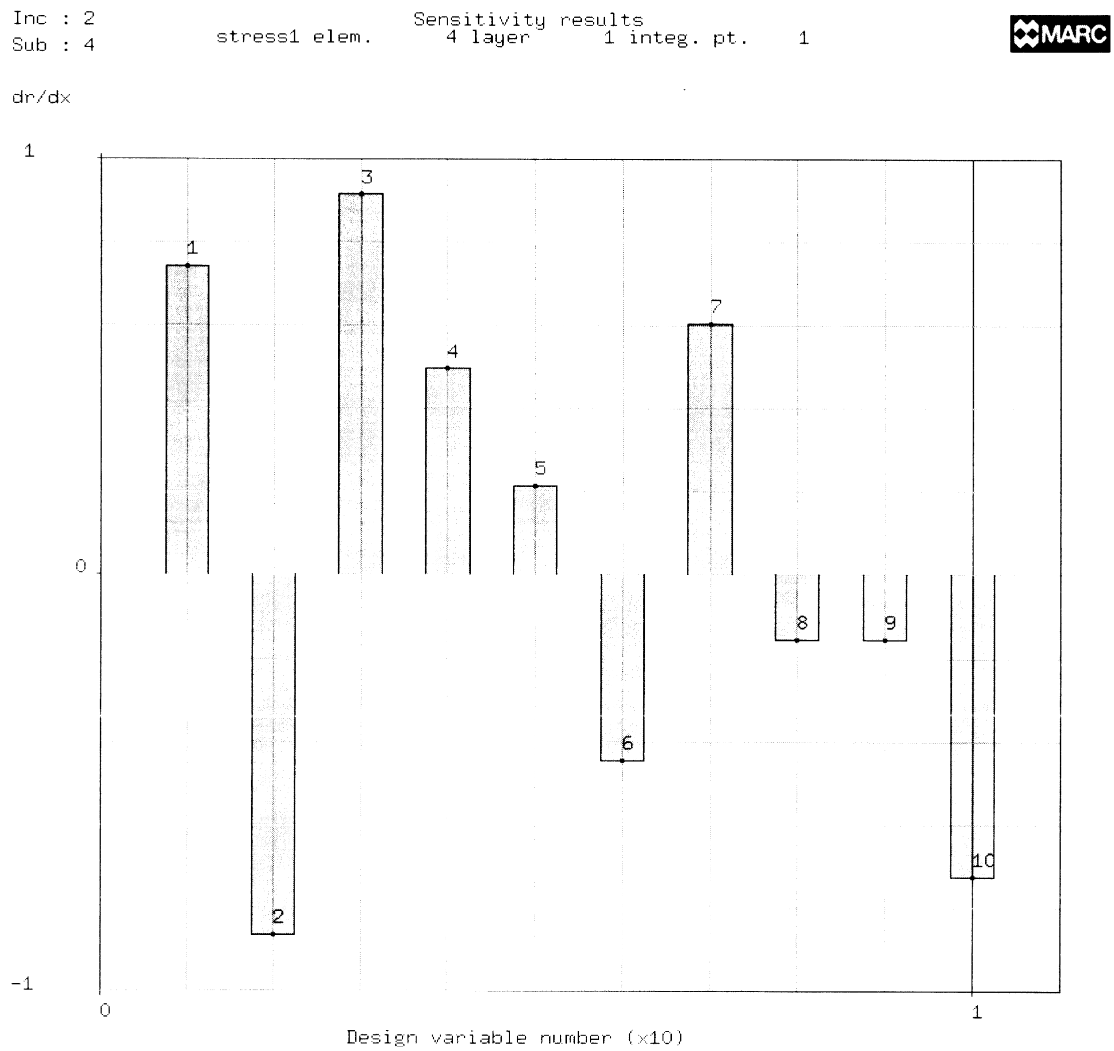

Figure 10.6-2 Gradient of Axial Stress in Element 4

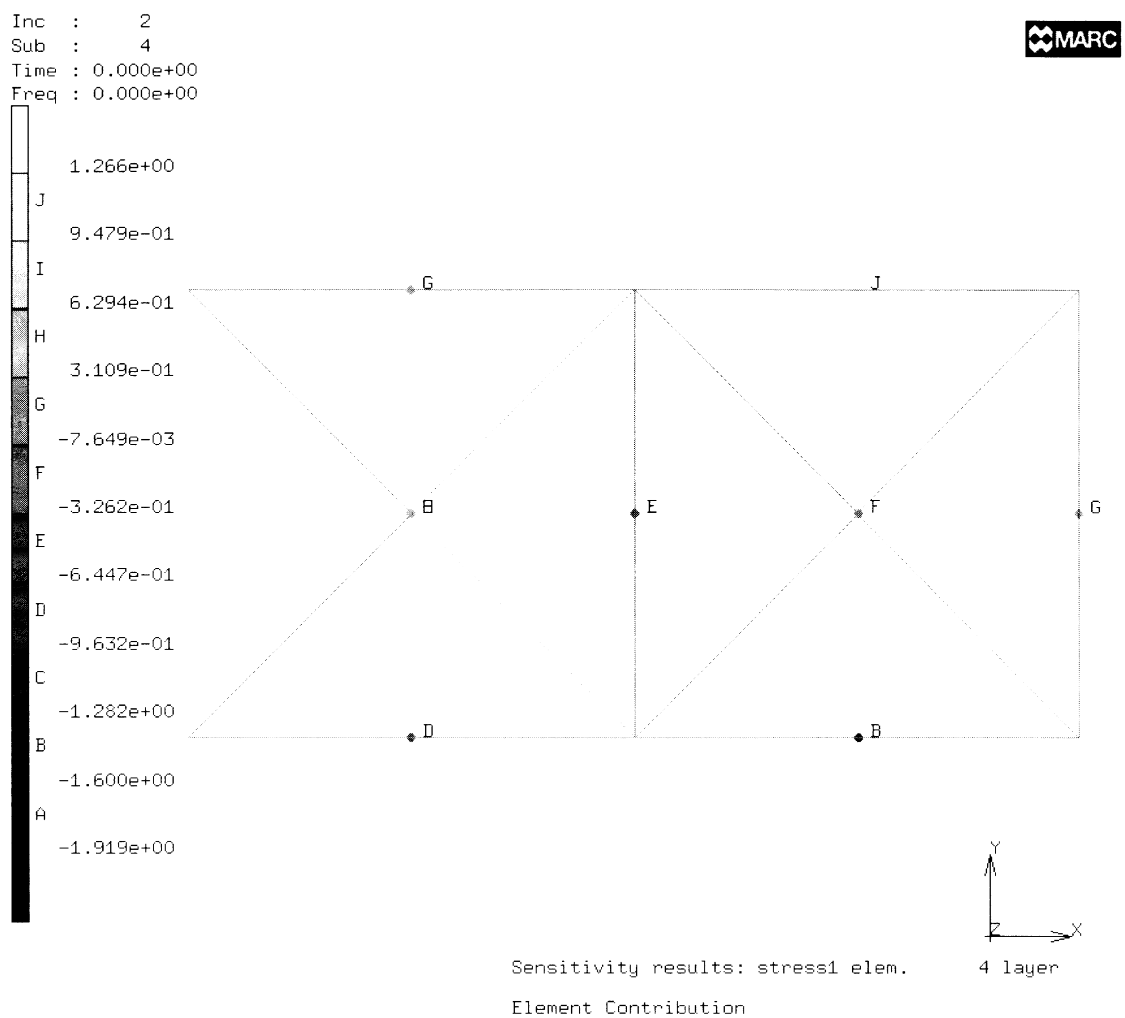

Figure 10.6-3 Element Contributions to Response of Figure 10.6-2

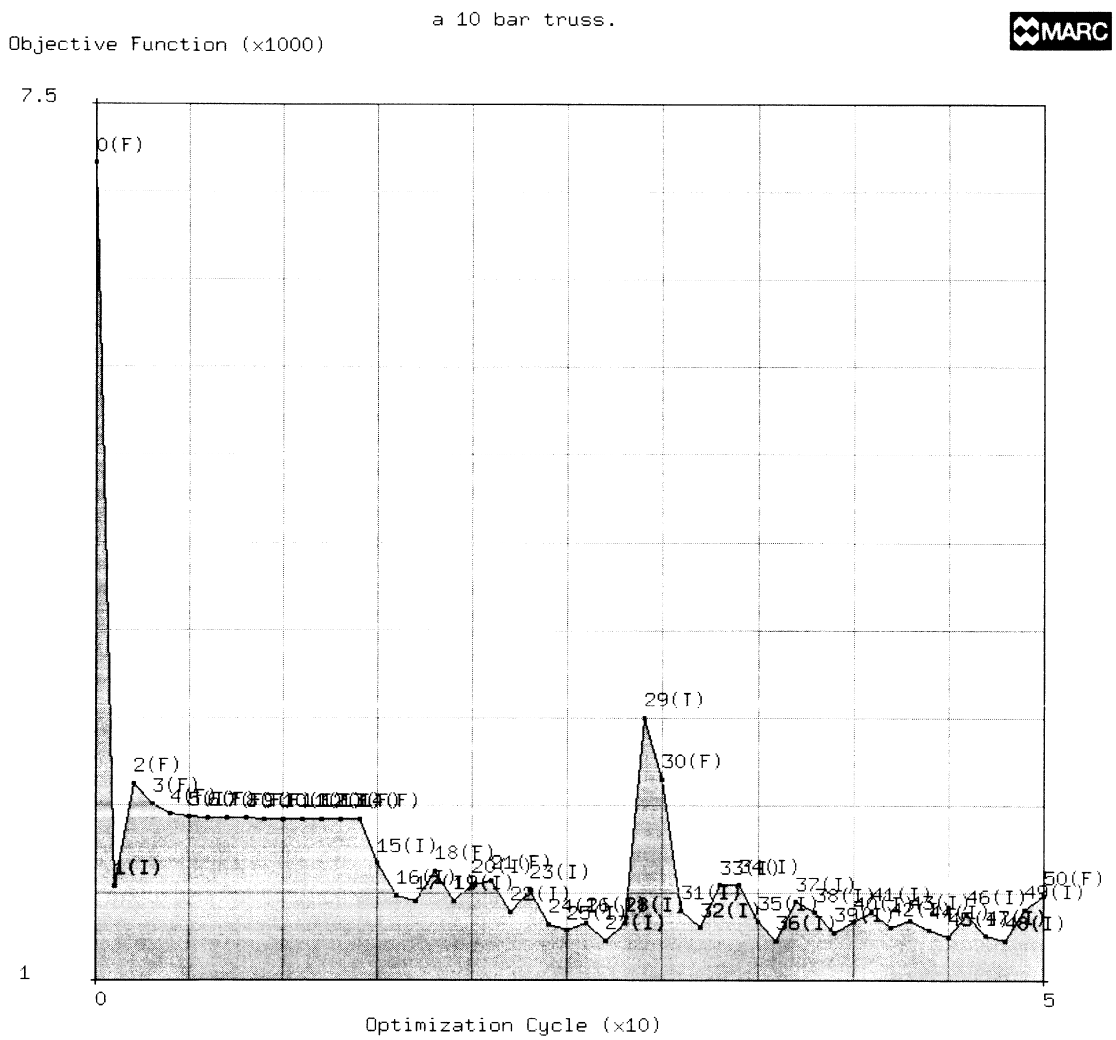

Figure 10.6-4 History Plot of the Objective Function

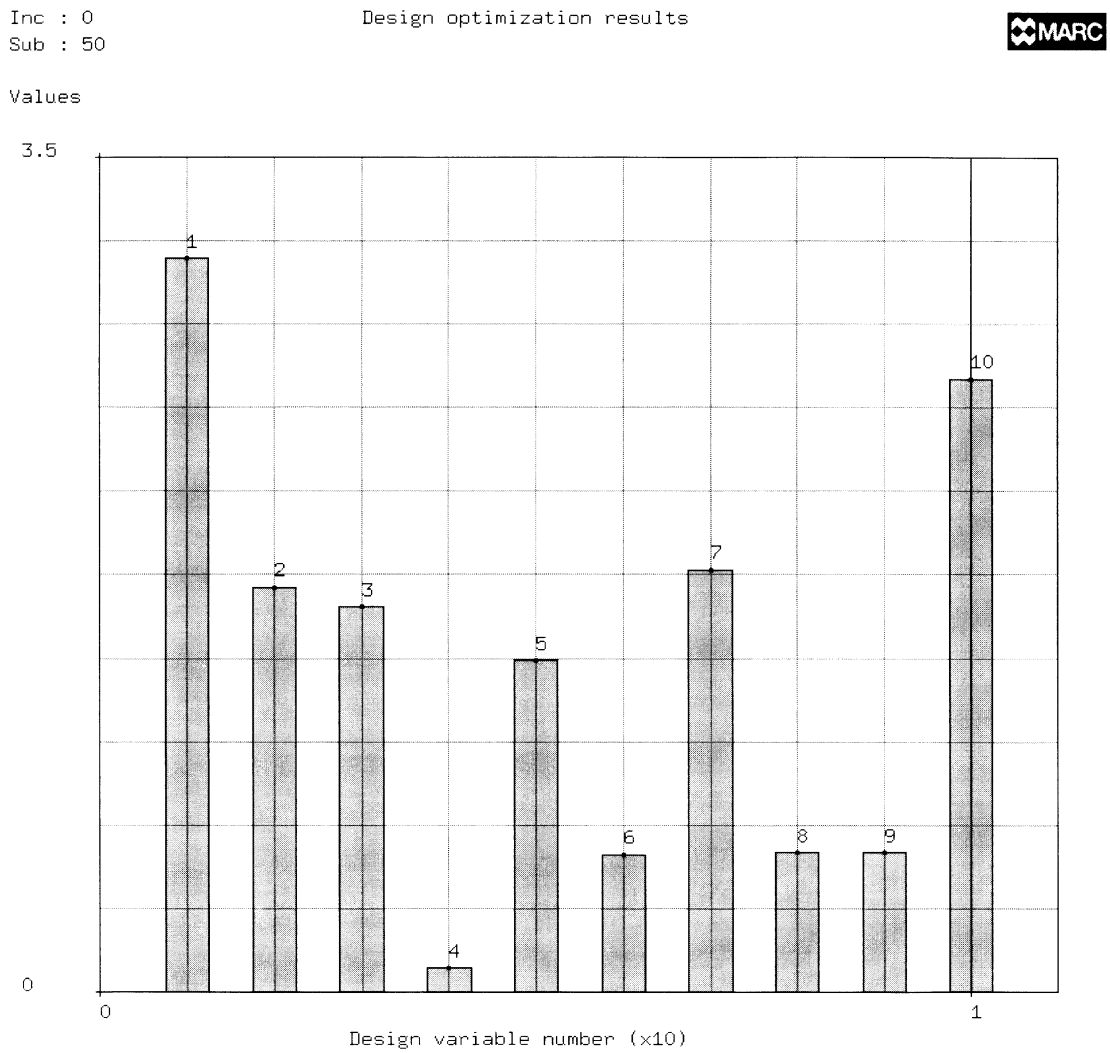

Figure 10.6-5 Design Variables at Best Feasible Design (Cycle 50)

10.7 Design Sensitivity Analysis and Optimization of an Alternator Mount using Element 14

This problem is geometrically similar to problem 10.1, except that the 3D frame model uses a different type of element which leads to a different choice of design variables. Differences also exist in material properties and, of course, in the geometry data.

Element)

The element used is type 14 with the default hollow circular section and two end nodes. The cross section of this element is defined by a wall-thickness and a mean radius.

Model

The 3D frame is modeled using 16 beam-column elements and 20 nodes. The columns are clamped at the base. The elements have arbitrary solid cross sections. Two masses are lumped in the middle of two horizontal beams at nodes 14 and 18 (Figure 10.7-1). Elements numbered 1 through 8 are the columns and the rest of the elements are the beams. All the elements have hollow circular cross sections. Column to beam connections are obtained through tying of separately numbered nodes.

Geometry

The geometry data consists of the wall-thickness and mean radius of the cross sections. For sensitivity analysis purposes, these are the same for all elements and are 0.1 and 10.0, respectively. For design optimization purposes, the lower- and upper-bounds are given by the DESIGN VARIABLES option.

Material Properties

The material is linearly elastic, homogeneous, and isotropic with a Young's modulus of 0.25×10^9, a Poisson's ratio of 0.349, and a mass density of 0.255×10^{-2}.

Loads

As in problem 10.1, there are three loadcases. The first consists of an eigenfrequency analysis. The second and third are static loadcases with point loads. All loadcases are the same as problem 10.1.

Design Variables and Objective Function

For elements 1 through 8 (the columns), the design variable is the wall-thickness linked over these elements (design variable 1). The lower- and upper-bounds for this first variable are 0.5 and 2.5, respectively. For elements 9 through 16 (the beams), the design variable is the mean radius of the cross section (design variable 2), again linked over the relevant elements. The lower- and upper-bounds for this second variable are 8.0 and 15.0, respectively.

The objective function for the problem is the total mass of the material used. Thus, sensitivity analysis obtains the gradient of this function and design optimization attempts to minimize it.

Design Constraints

The constraints are imposed on generalized stresses, a translation and a rotation, and eigenfrequencies. For demonstration purposes, all three generalized stress constraints are on element 13 for loadcase two, and the two displacement constraints are on nodes 15 and 19, respectively. The first eigenfrequency constraint is on the fundamental mode. The second eigenfrequency constraint is on the difference between the frequencies of the first and second modes of free vibration.

Results

The design sensitivity and design optimization cases are run as separate jobs, with the data files e10x7a.dat and e10x7b.dat, respectively. Figure 10.7-2 shows the gradient of the first eigenfrequency with respect to the two design variables. Figure 10.7-3 is an element values plot showing, on the finite element model, the element contributions to the response quantity in question. Figure 10.7-4 shows the change in the objective function with the optimization cycle in the form of a path plot. Figure 10.7-5 is a bar chart showing the values of the design variables at the best design obtained. It is noted that no feasible design is found for this problem. However, the normalized most critical value is -0.004 at the best design (cycle 7), indicating that this design is very close to being feasible. In comparison, the design at the starting vertex (cycle 0) had a most critical normalized constraint value of -0.175, indicating severe infeasibility.

Parameters, Options, and Subroutines Summary

Listed below are the options used in example e10x7a.dat:

Parameters	Model Definition Options	History Definition Options
DESIGN SENSITIVITY	CONNECTIVITY	CONTINUE
DYNAMIC	COORDINATES	MODAL SHAPE
ELEMENTS	DESIGN DISPLACEMENT CONSTRAINTS	POINT LOAD
END	DESIGN FREQUENCY CONSTRAINTS	
SIZING	DESIGN OBJECTIVE	
TITLE	DESIGN STRESS CONSTRAINTS	
	DESIGN VARIABLES	
	END OPTION	
	FIXED DISP	
	GEOMETRY	
	ISOTROPIC	
	MASSES	
	POINT LOAD (dummy)	
	POST	
	TYING	

Listed below are the options used in example e10x7b.dat:

Parameters	Model Definition Options	History Definition Options
DESIGN OPTIMIZATION	CONNECTIVITY	CONTINUE
DYNAMIC	COORDINATES	MODAL SHAPE
ELEMENTS	DESIGN DISPLACEMENT CONSTRAINTS	POINT LOAD
END	DESIGN FREQUENCY CONSTRAINTS	
SIZING	DESIGN OBJECTIVE	
TITLE	DESIGN STRESS CONSTRAINTS	
	DESIGN VARIABLES	
	END OPTION	
	FIXED DISP	
	GEOMETRY	
	ISOTROPIC	
	MASSES	
	POINT LOAD (dummy)	
	POST	
	TYING	

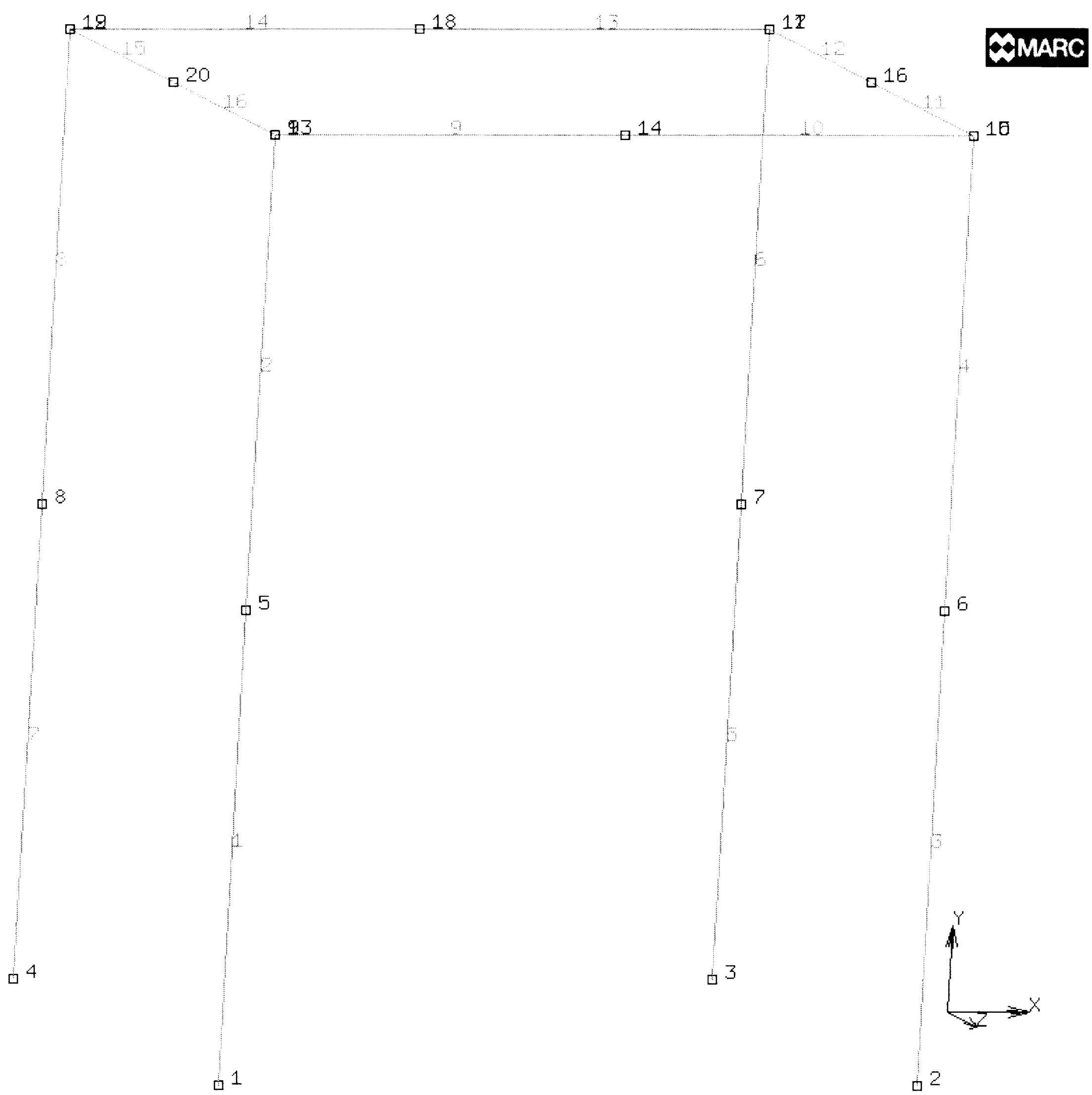

Figure 10.7-1 Alternator Mount Frame Model using Element 14

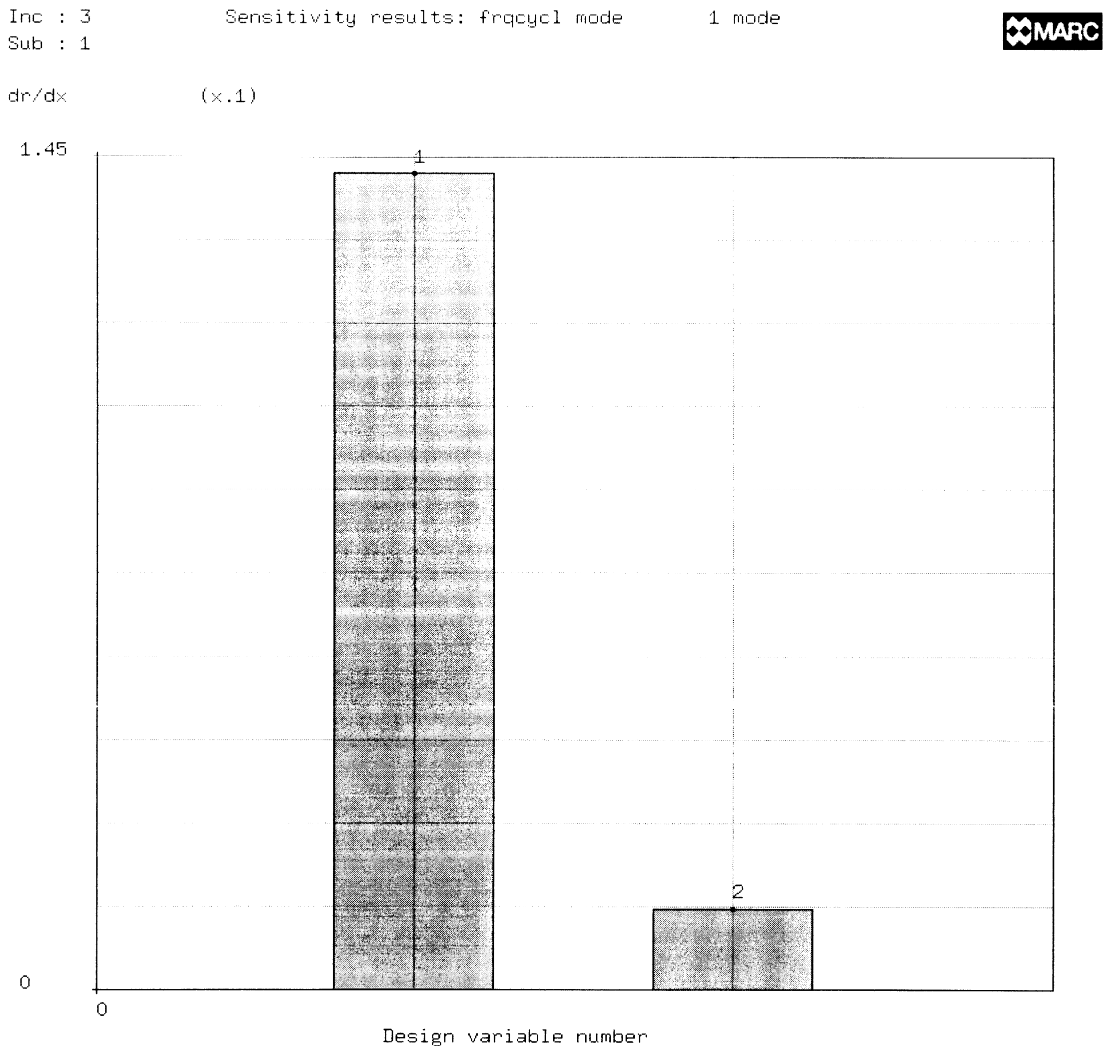

Figure 10.7-2 Gradient of the First Eigenfrequency with Respect to Design Variables

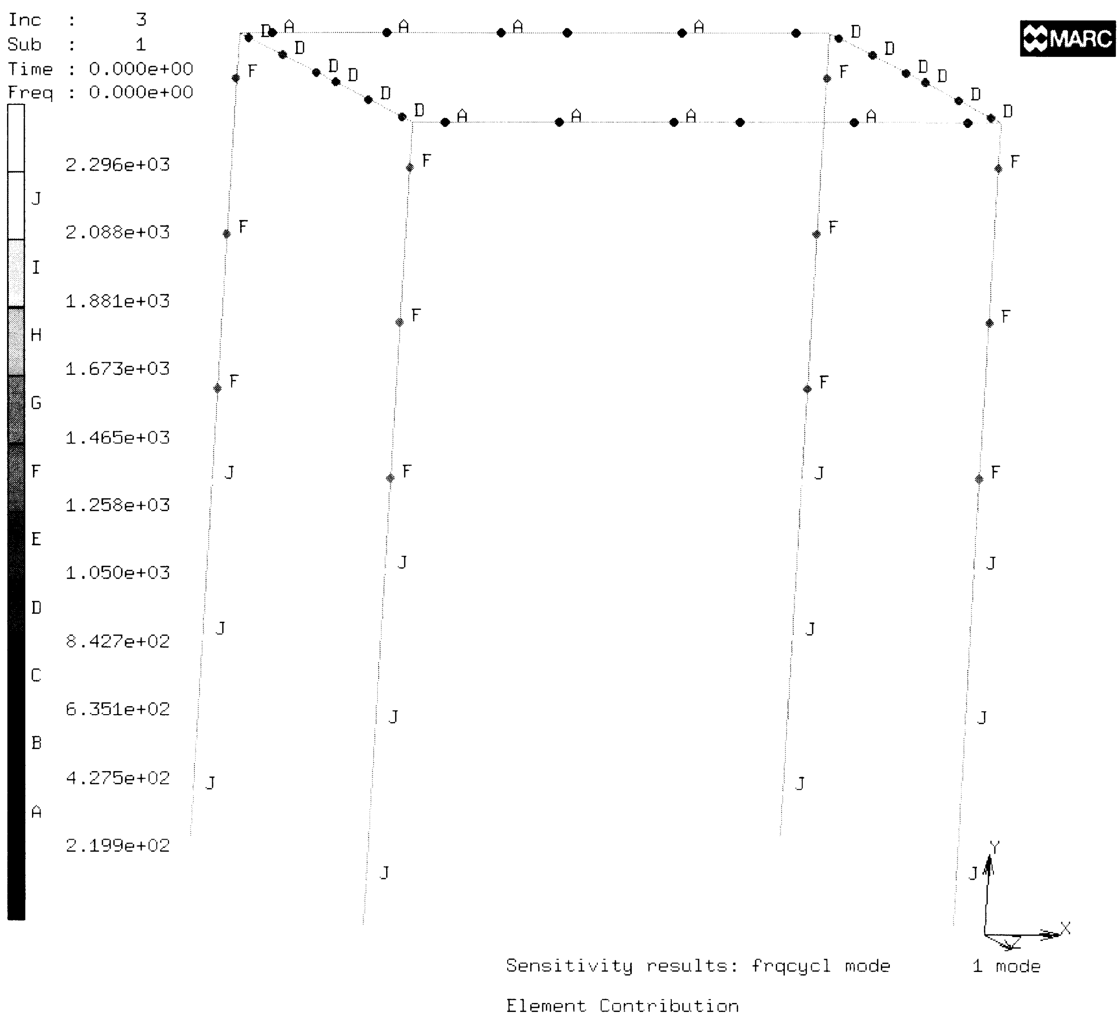

Figure 10.7-3 Element Contributions to Response of Figure 10.7-2

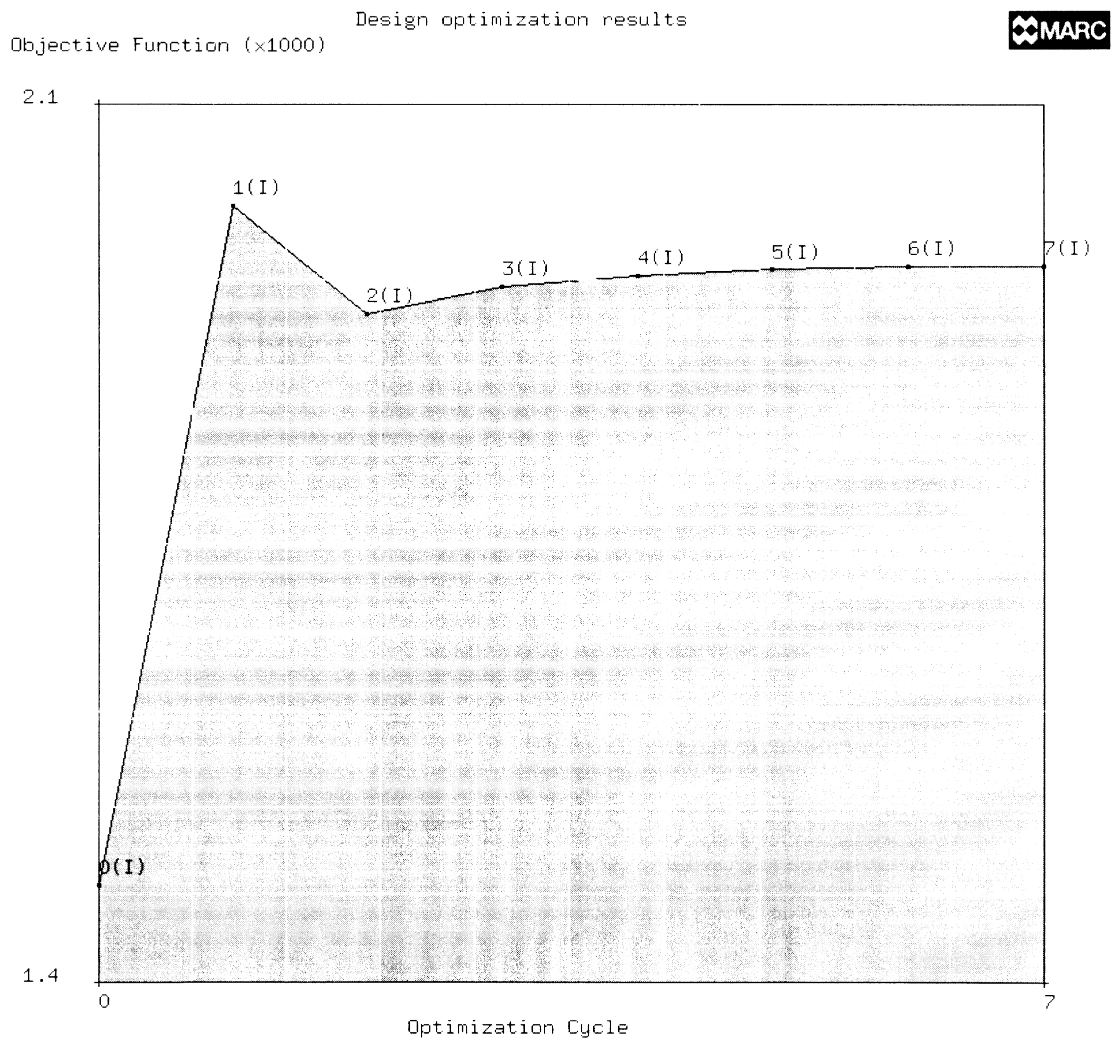

Figure 10.7-4 History Plot of the Objective Function

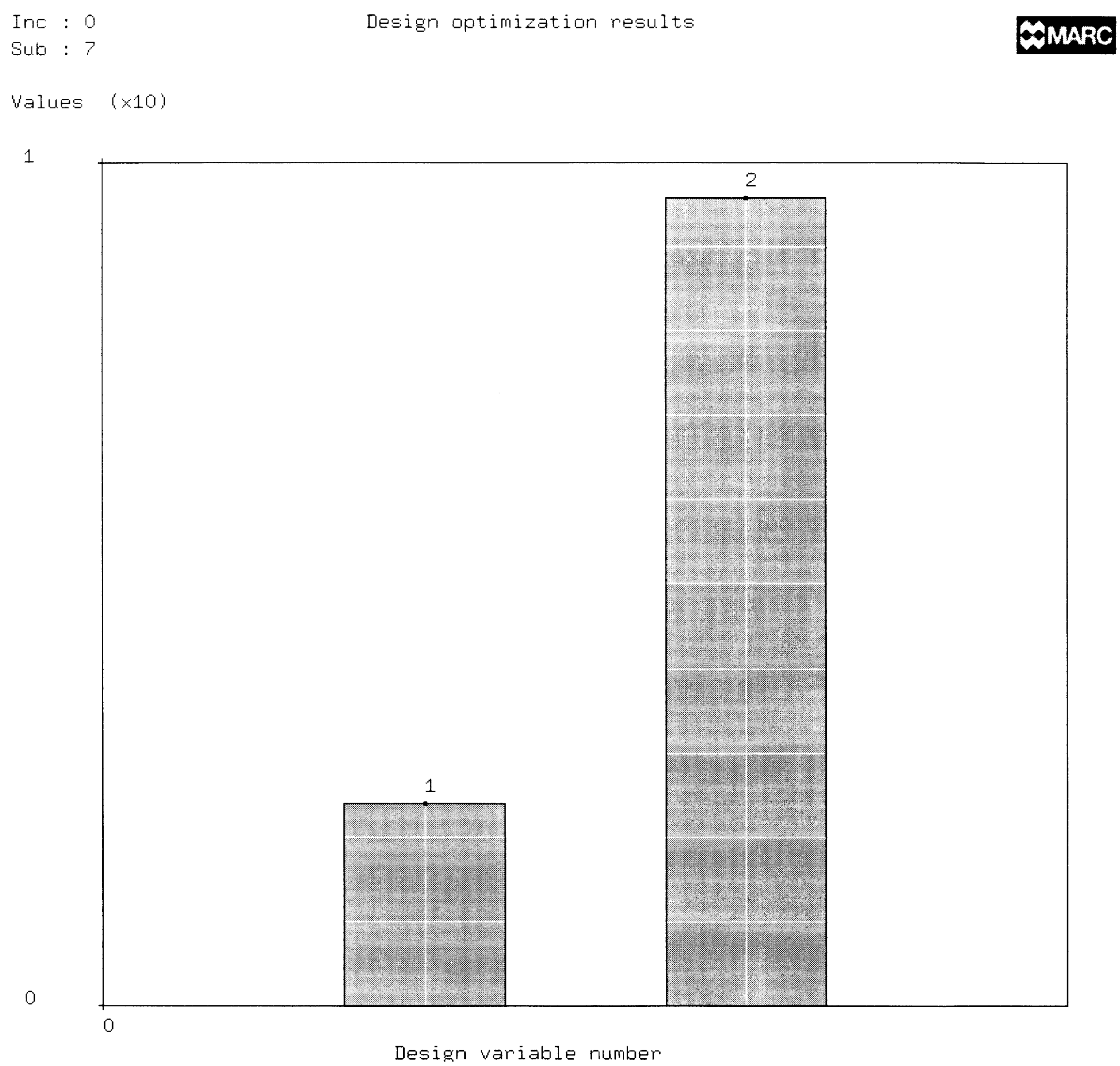

Figure 10.7-5 Design Variables at Best Design (Cycle 7)